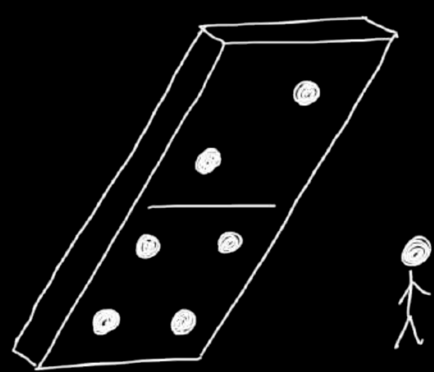
Metaphysics (MetaPhysics? Meta Physics?)

Idea: sculpture of giant dominoes made up of dice

(4/27/2021)

- inspired by clip I saw of Tony Cragg's work w/ dice
- want this to be huge, looming over the viewer, mounted in a teetering pose



- Standard dominoes are 1 7/8" × 15/16" × 1/4"
  - Say I want this to be 12' tall. The dimensions would be 144" × 72" × 19.2"

    = 12' × 6' × 19.2"

    = 365.76cm × 182.88cm × 48.768cm

  - For 10×10×10mm dice:
    - Each is 1000 mm^3
    - To fill volume: 3,262,100.73 dice!

- To cover surface:
    - Each big face: 66,890 dice
    - Each long side (excluding edges already taken care of by big faces): 17,106 dice
    - Each short side (excluding edges already taken care of by big faces and long sides): 8,459 dice
    - TOTAL: 184,910 dice!
    - Even at cheap China rates (~$0.03/each), that's $5,547.30!

See spreadsheet I made (in Numbers)

- could do something similar with coins...

Title ideas:

- Metaphysics

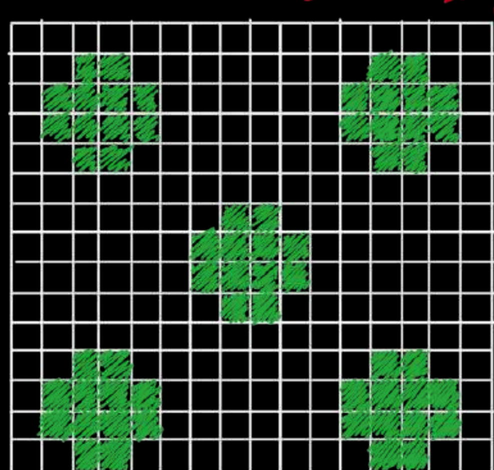
- Unmoved mover
- First cause / Final cause
- Probably inevitable / Probable inevitability
- Inevitable probability

https://m.alibaba.com/product/62184621859/Wholesale-12mm-14mm-16mm-18mm-20mm.html?__detailProductImg=https%3A%2F%2Fs.alicdn.com%2F%40sc01%2Fkf%2FHTB1FW.UeoGF3KVjSZFvq6z_nXXa2.jpg_200x200.jpg

Email these people to learn about how universities pick art installations: https://news.stanford.edu/2021/04/21/alicja-kwade-site-specific-installation-stanford-science-engineering-quad-suggests-alternate-realities/

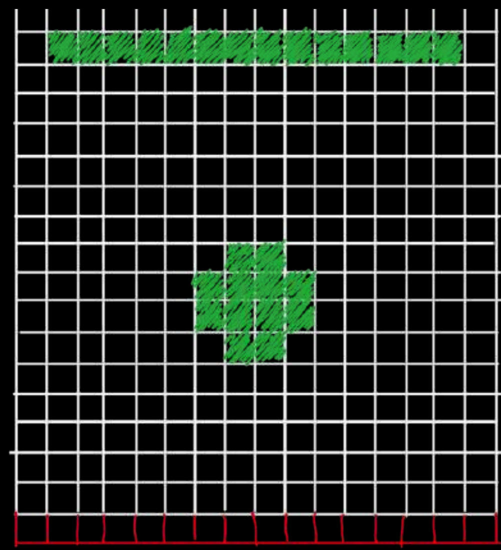
9/15/2021:

- Use a Minecraft pixel circle generator to easily see what circles made of squares look like
- Suppose the diameter of a dot is about 1/8 the height of a domino face
- Suppose I make 4x4 dice circles for the dots (in a small model)  this is about as small as I can go, where they still look like circle
- Then, the dimensions of the model in dice should be:
  - dot: 4 x 4
  - height: 3~~2~~3
  - width: 16
  - depth: ~~4~~ 6
    - to better match dimensions of dominoes I bought
  - 1 between dot rows and columns
  - 1 between upper and lower edges and dot rows, and left/right edges and dot colums
  - Should add one more row to accomodate middle line



16 x 0.63" 33 x 0.63"

10.08" x 20.79"



- Total dice needed:
  - to fill volume: 2,112
  - to cover surface:

main faces: (33 × 16 × 2)

long sides: + (33 × ~~2~~ 4 × 2)

short sides: + (14 × ~~2~~ 4 × 2)

= ~~1,244~~ 1,388

[14 + 12 * (# dots)] black

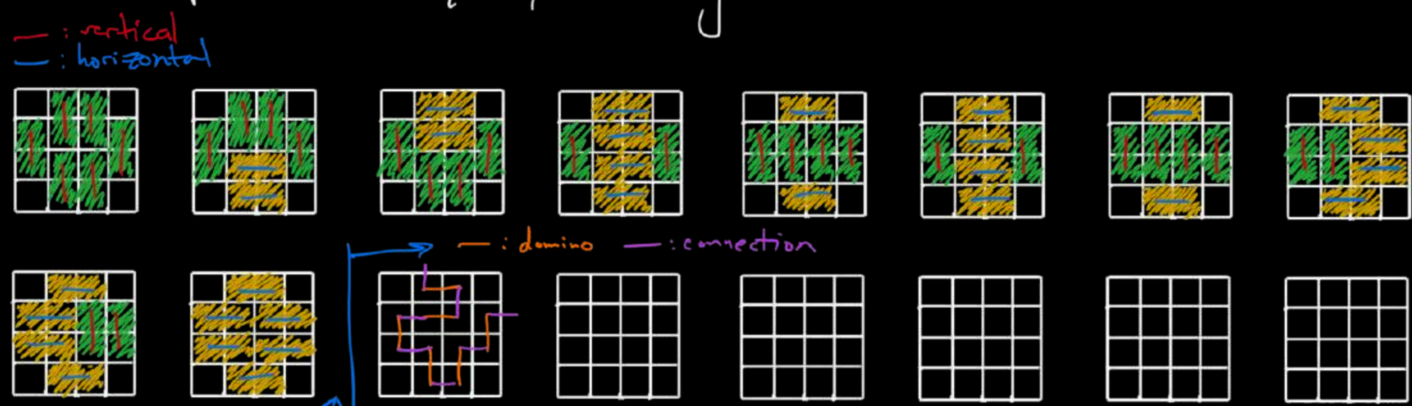
rest white

1,2 → 50b, ~~1194w~~ 1288
1,3 → 62b, ~~1182w~~ 1276
1,4 → 74b, ~~1170w~~ 1264
1,5 → 86b, ~~1158w~~ 1252
1,6 → 98b, ~~1146w~~ 1240
2,3 → 74b, ~~1170w~~ 1264
2,4 → 86b, ~~1158w~~ 1252
2,5 → 98b, ~~1146w~~ 1240
2,6 → 110b, ~~1134w~~ 1228
3,4 → 98b, ~~1146w~~ 1240
3,5 → 110b, ~~1134w~~ 1228
3,6 → 122b, ~~1122w~~ 1216
4,5 → 122b, ~~1122w~~ 1216
4,6 → 134b ~~1110w~~ 1204
5,6 → 146b, ~~1098w~~ 1292

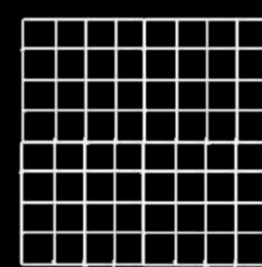
- # black dice: 12 × (# dots) +14! (for middle line)

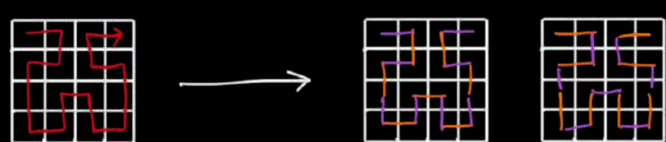
FOR DICE OF DOMINOES:

- Suppose I use 4×4 circles of dominoes for the dice dots
- The pattern/structure would be the same as above, except for the extra row
- Possible 4×4 tilings:

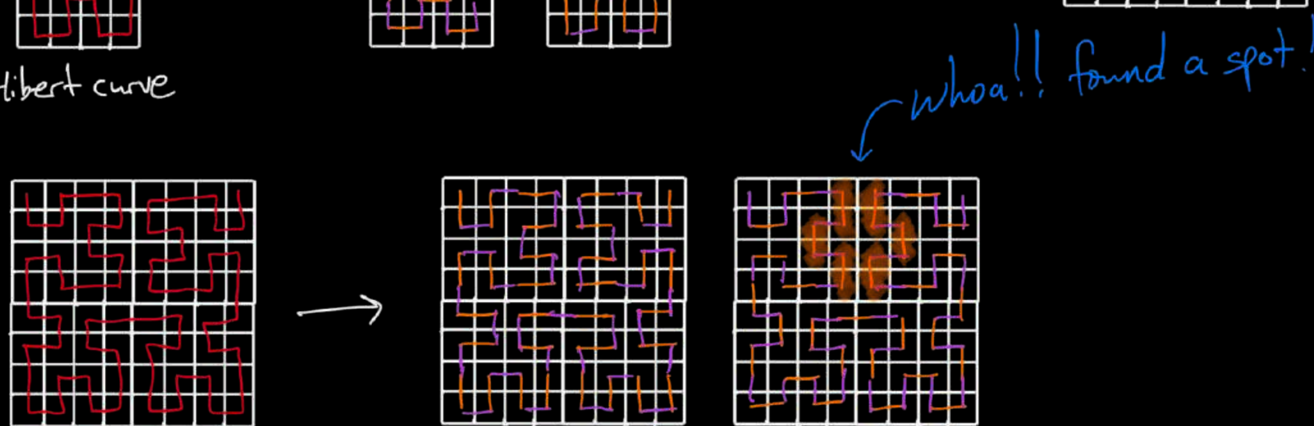


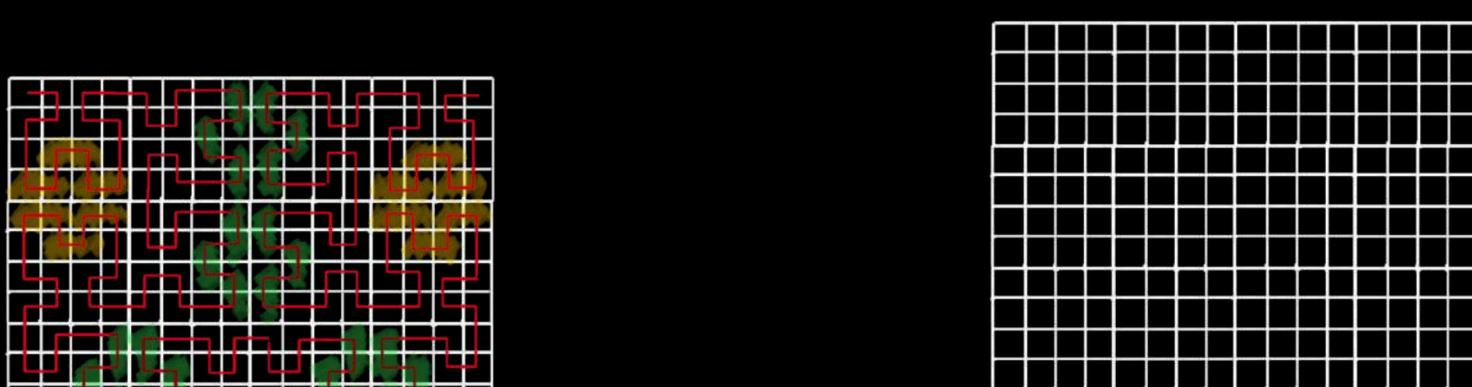
Connections to space filling curves...

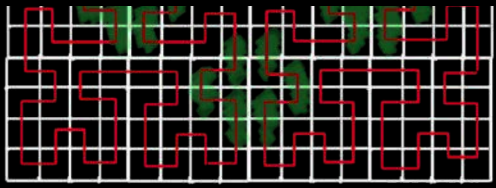


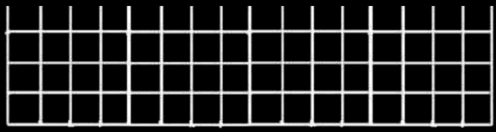


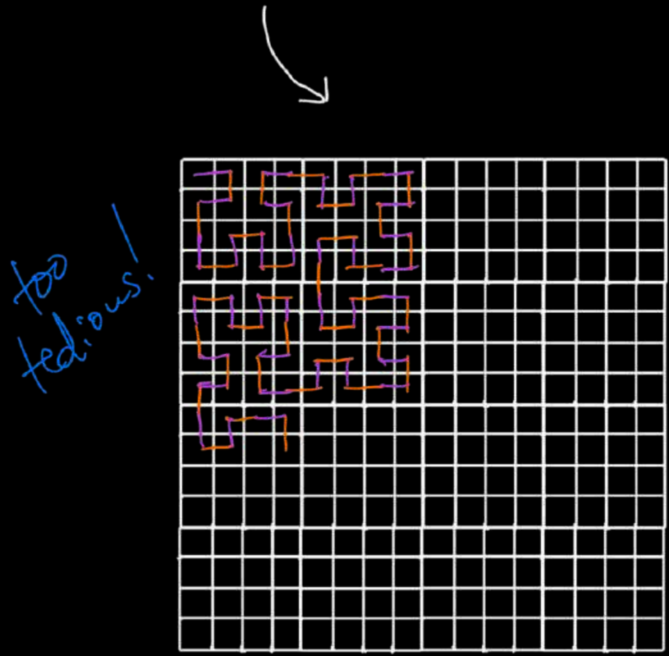
Hibert curve

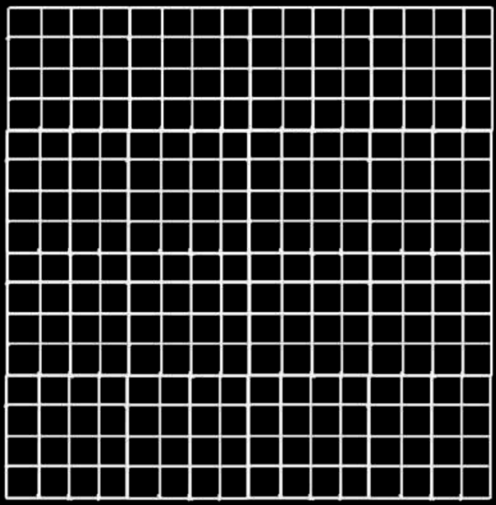






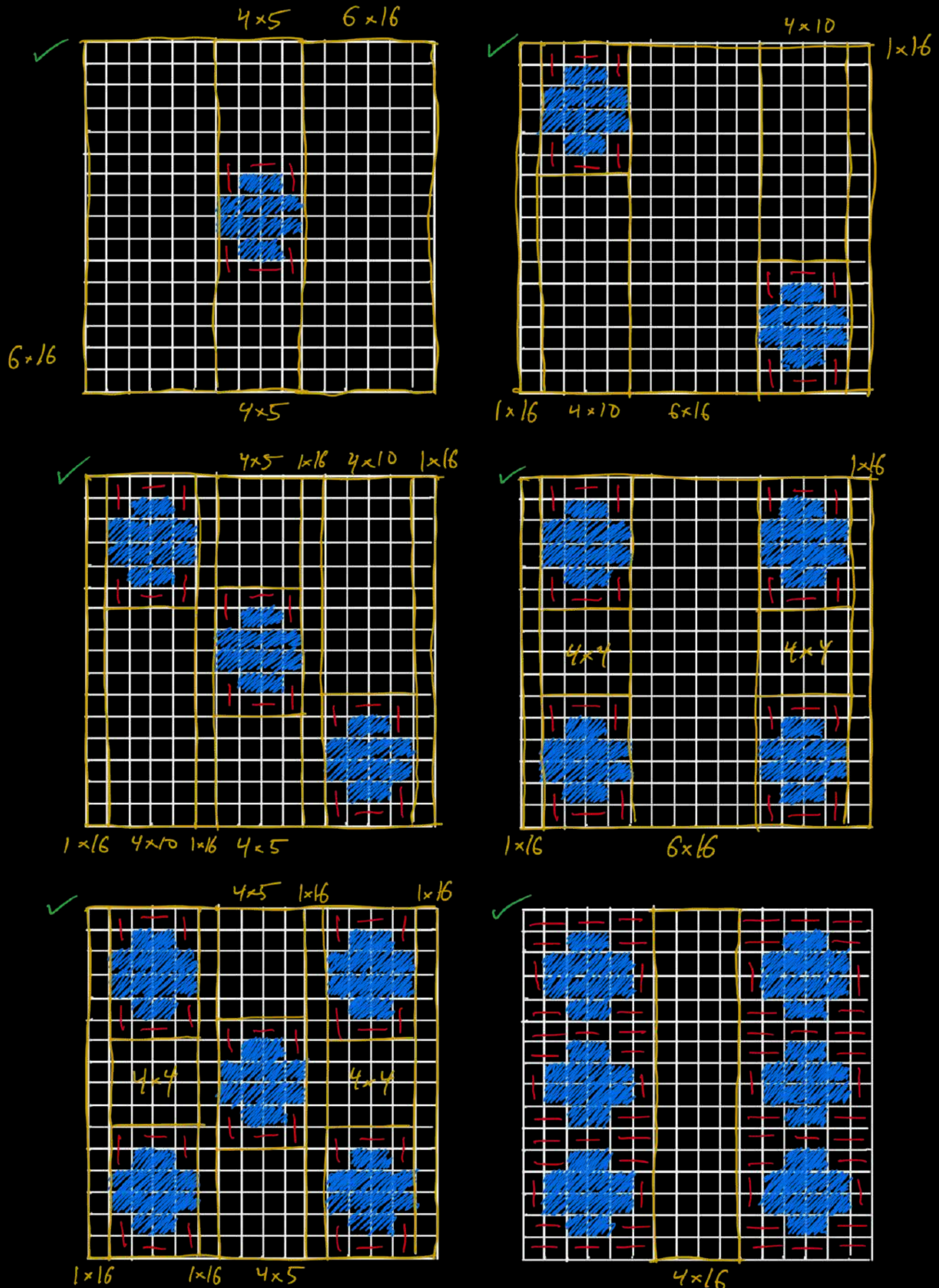






see further below for image of Hibert curves of higher order, with my highlights...

- Let me make sure I could tile around the dots...



- I know I can tile any m×n

rectangle when $m \times n$ is even

- So, a good strategy is tiling around the dots until I have nothing but such regions left

- Total dominoes needed:
  - to cover surface:

$$[(16 \times 16) \div 2] \times 6$$

$$+ (16 \div 2) \times 8$$

← to line edges, to cover up lack of good fit

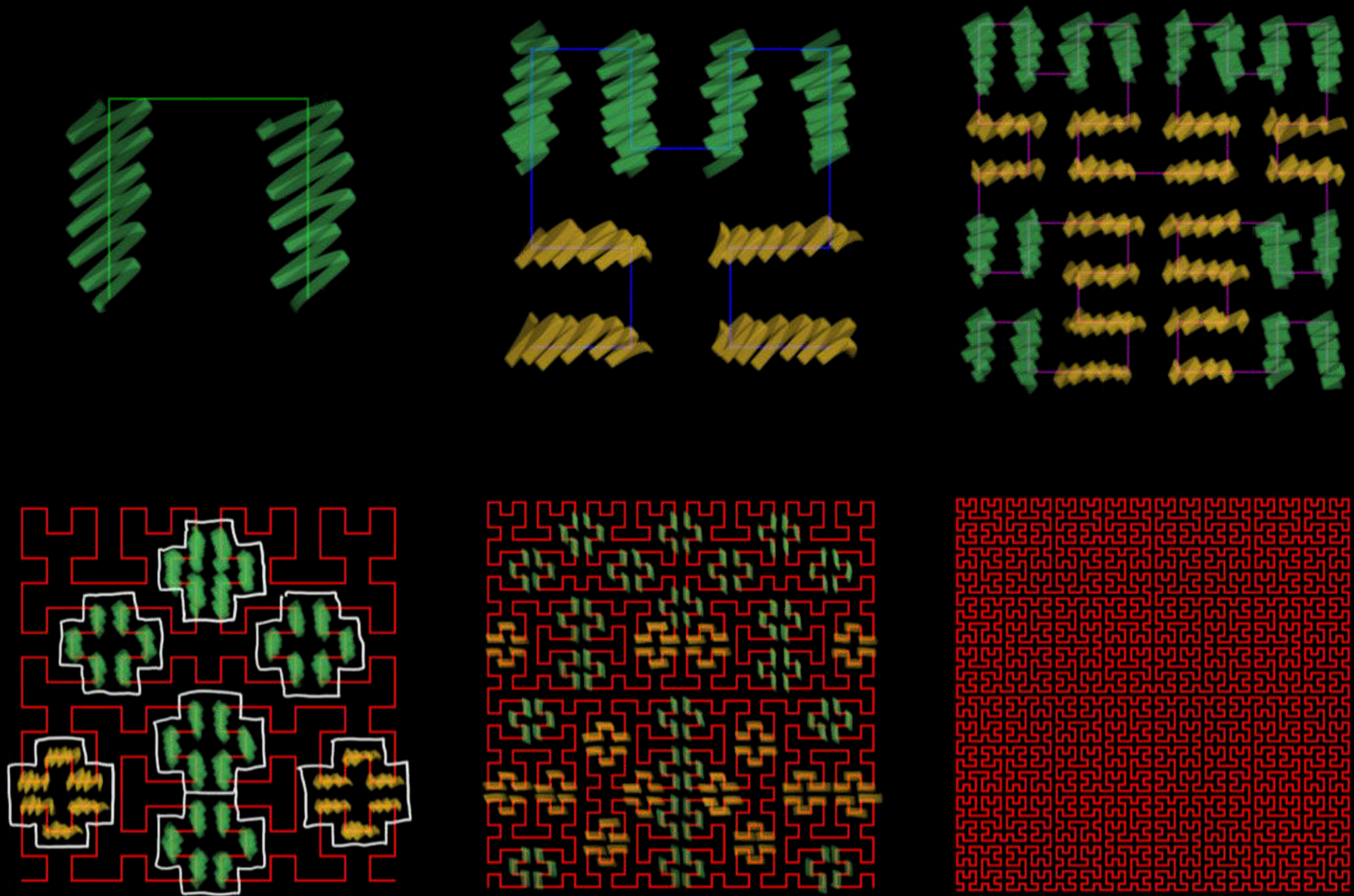
$$= 832$$

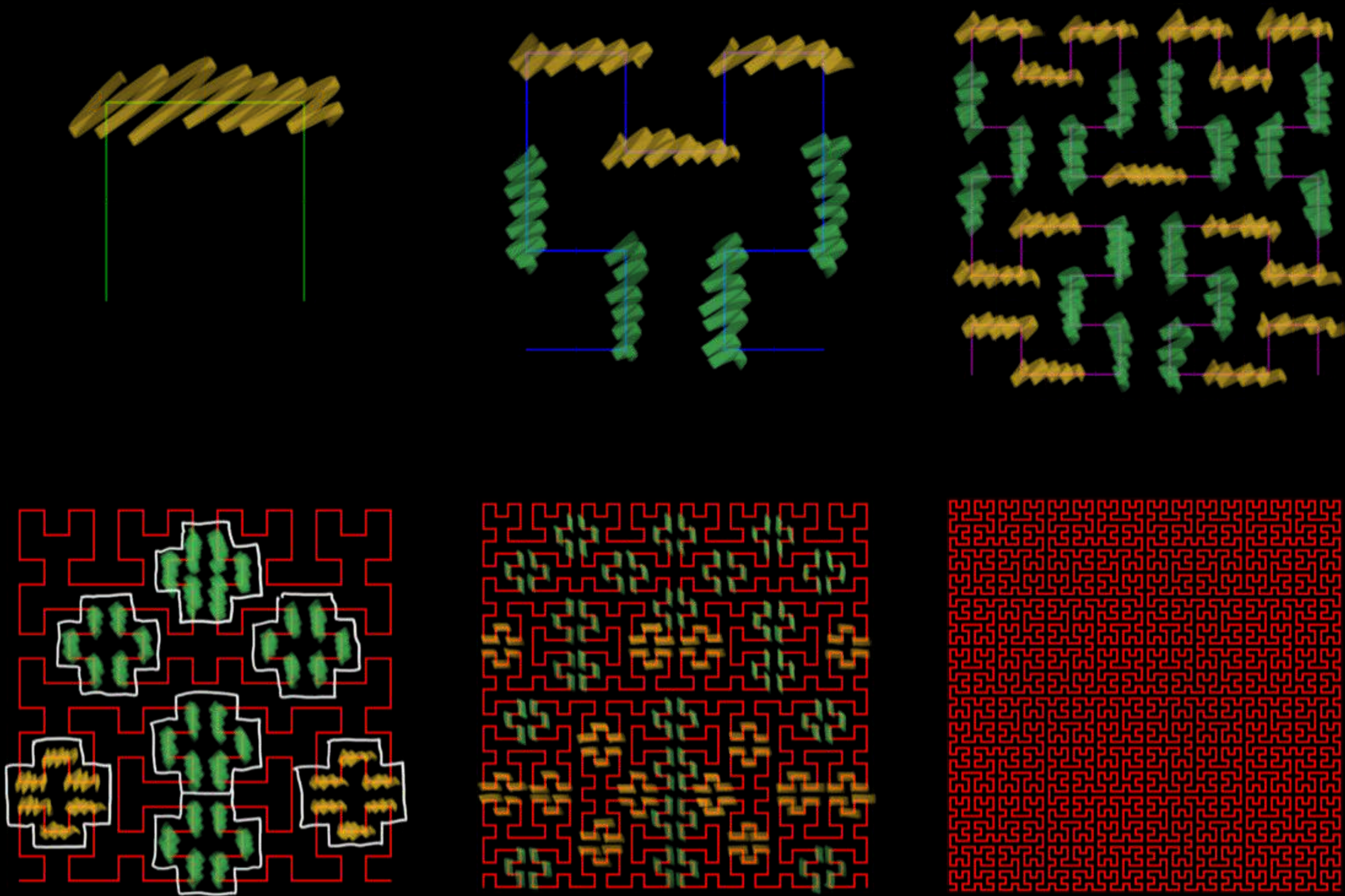
126 black, 706 white

OR w/o line edges:

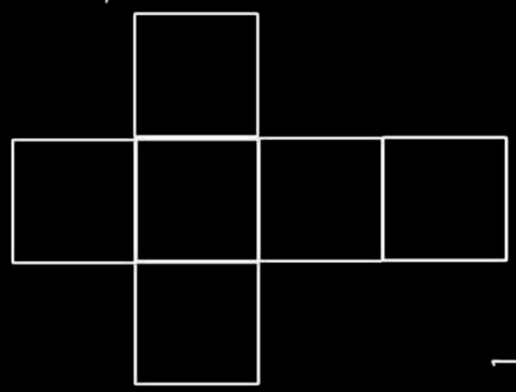
768

126 black, 642 white

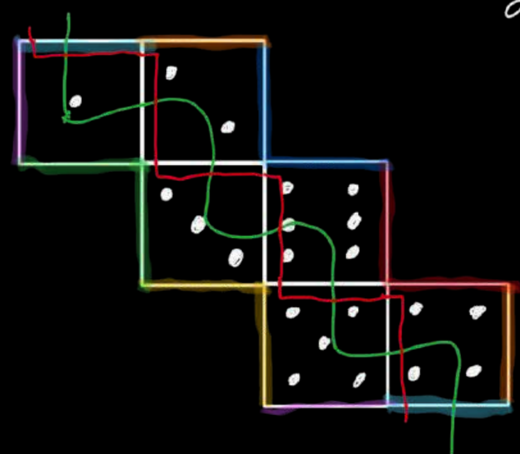




- Ways to unfold a cube:



- See paper for how to start and end Hilbert type curve at any point on square ("Hilbert-Type Space-Filling Curves" by Nicholas J. Rose)



- Standard domino set:

[[0,0]

28

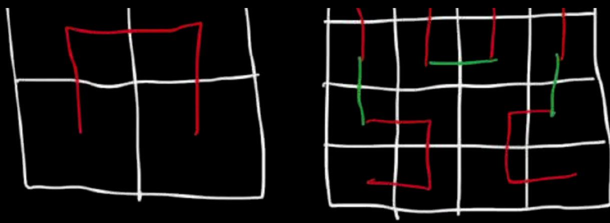
[0,1], [1,1]
[0,2], [1,2], [2,2]
[0,3], [1,3], [2,3], [3,3]
[0,4], [1,4], [2,4], [3,4], [4,4]
[0,5], [1,5], [2,5], [3,5], [4,5], [5,5]
[0,6], [1,6], [2,6], [3,6], [4,6], [5,6], [6,6]

21

zeroes : 8
ones : 8
twos : 8
threes : 8
fours : 8
fives : 8
sixes : 8

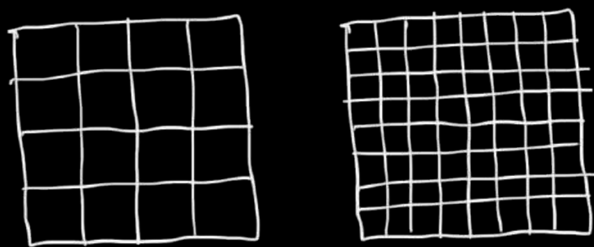
- Want to figure out how to design my <u>own</u> space-filling curve. Let's start by better understanding the Hilbert construction:

doesn't seem

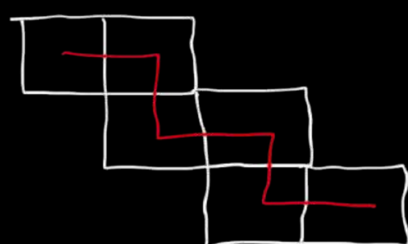


Hmm [illegible] alternative to be any since this is (isomorphic to) the only way to tile a 2x2 grid

- Now, my own ideas...

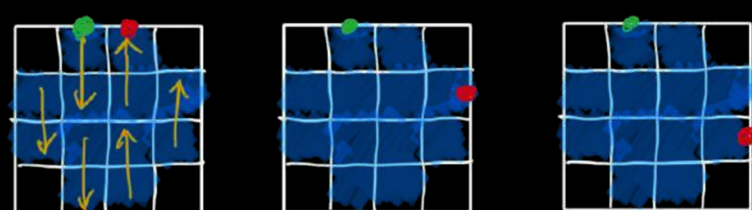


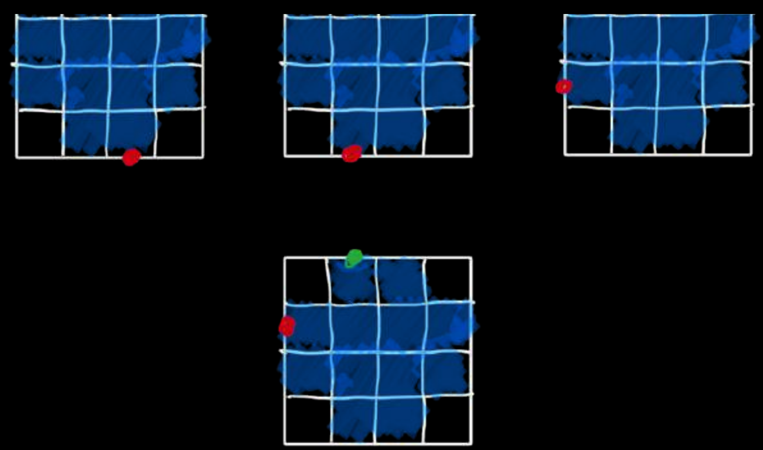
- What if I make the fundamental unit not a 2x2 grid but the same shape as the unfolded cube?



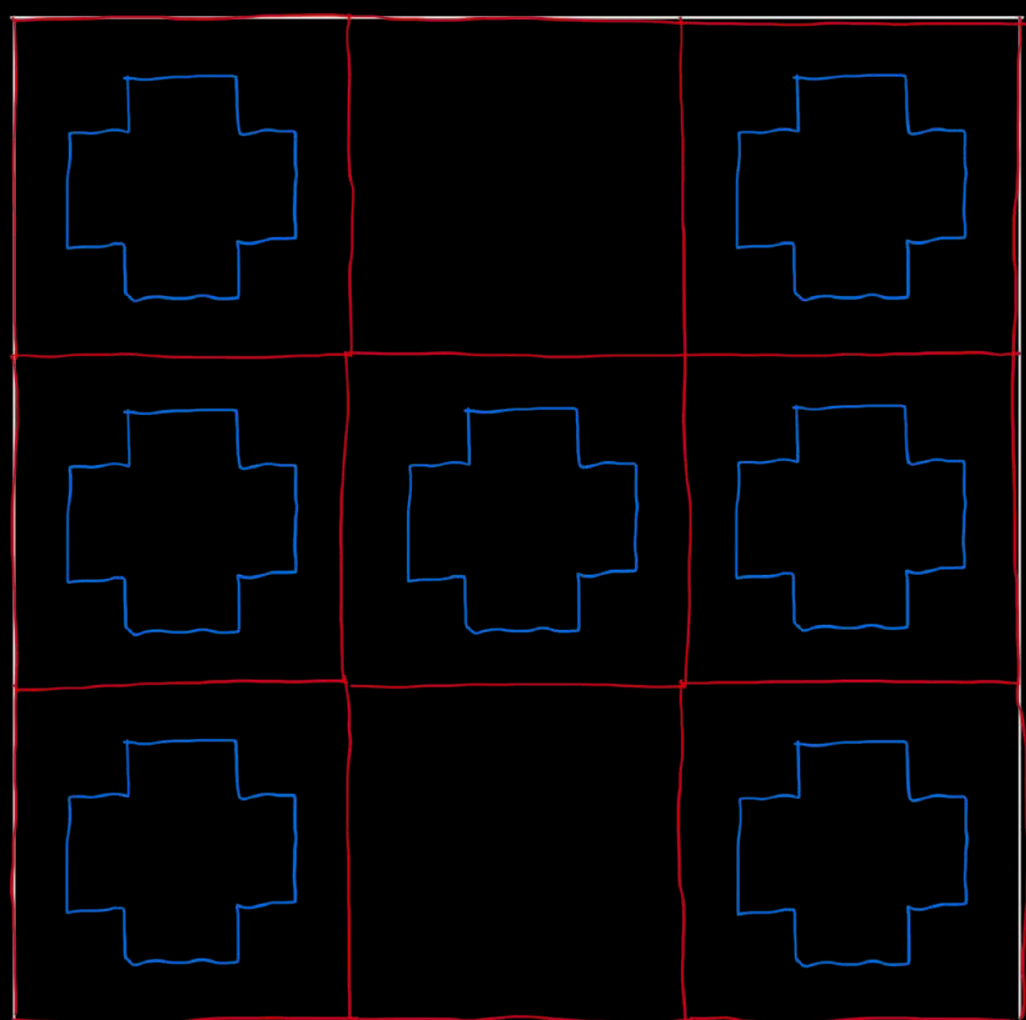
Don't think this works b/c it's not self-similar

- How to go "in and out" of the dots:

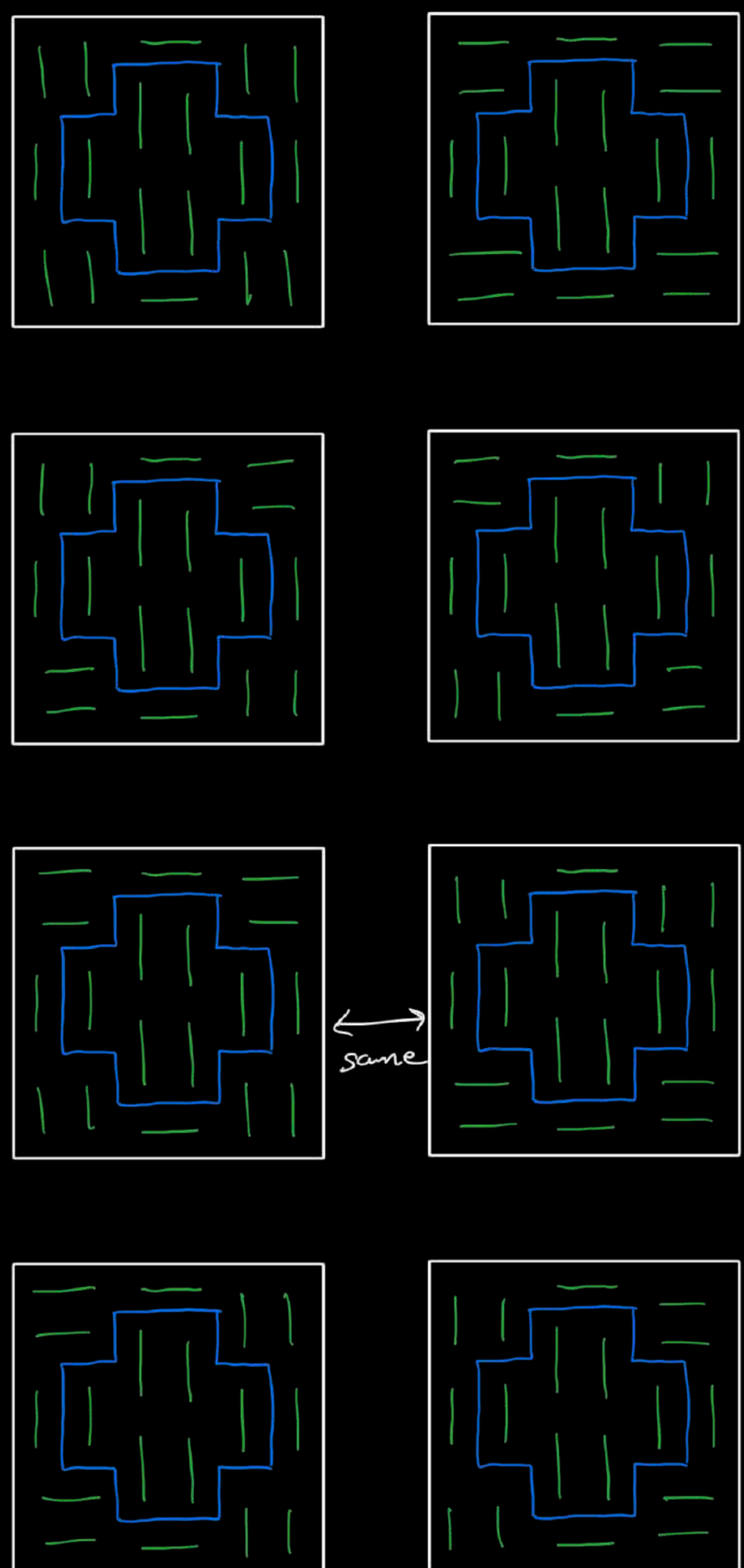


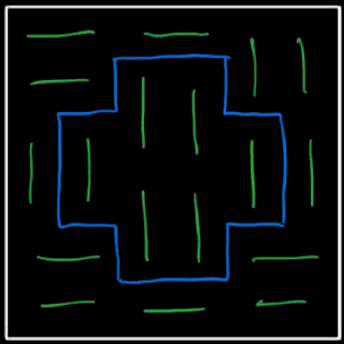
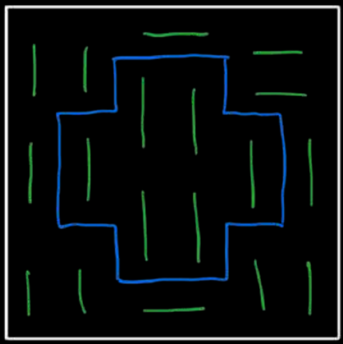
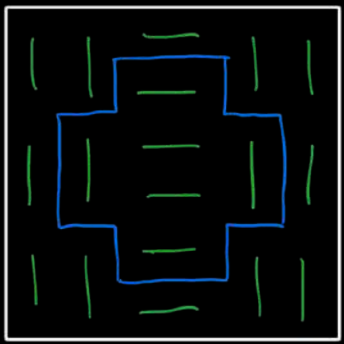
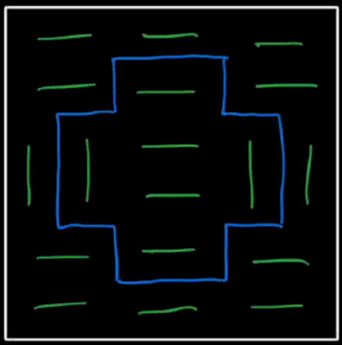
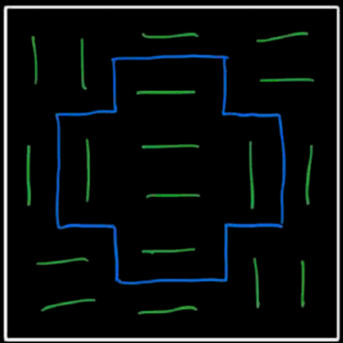
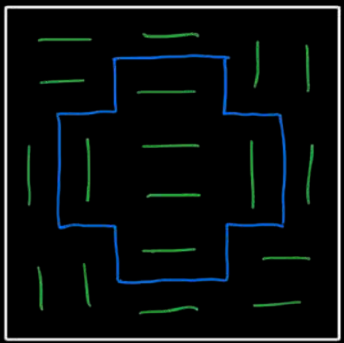


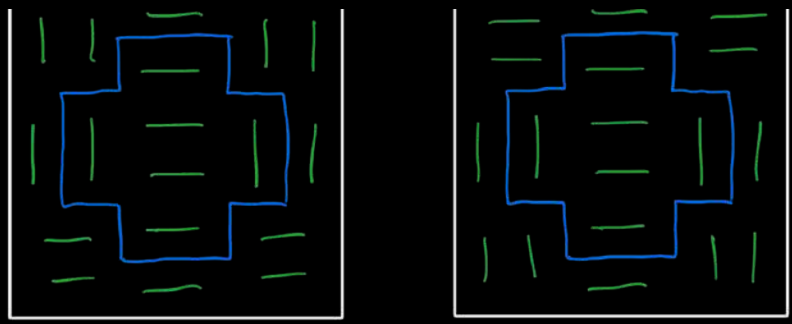
- What if I still used 4×4 dots (which exist in 6×6 tiled grids) but expanded the dice face to 18×18?



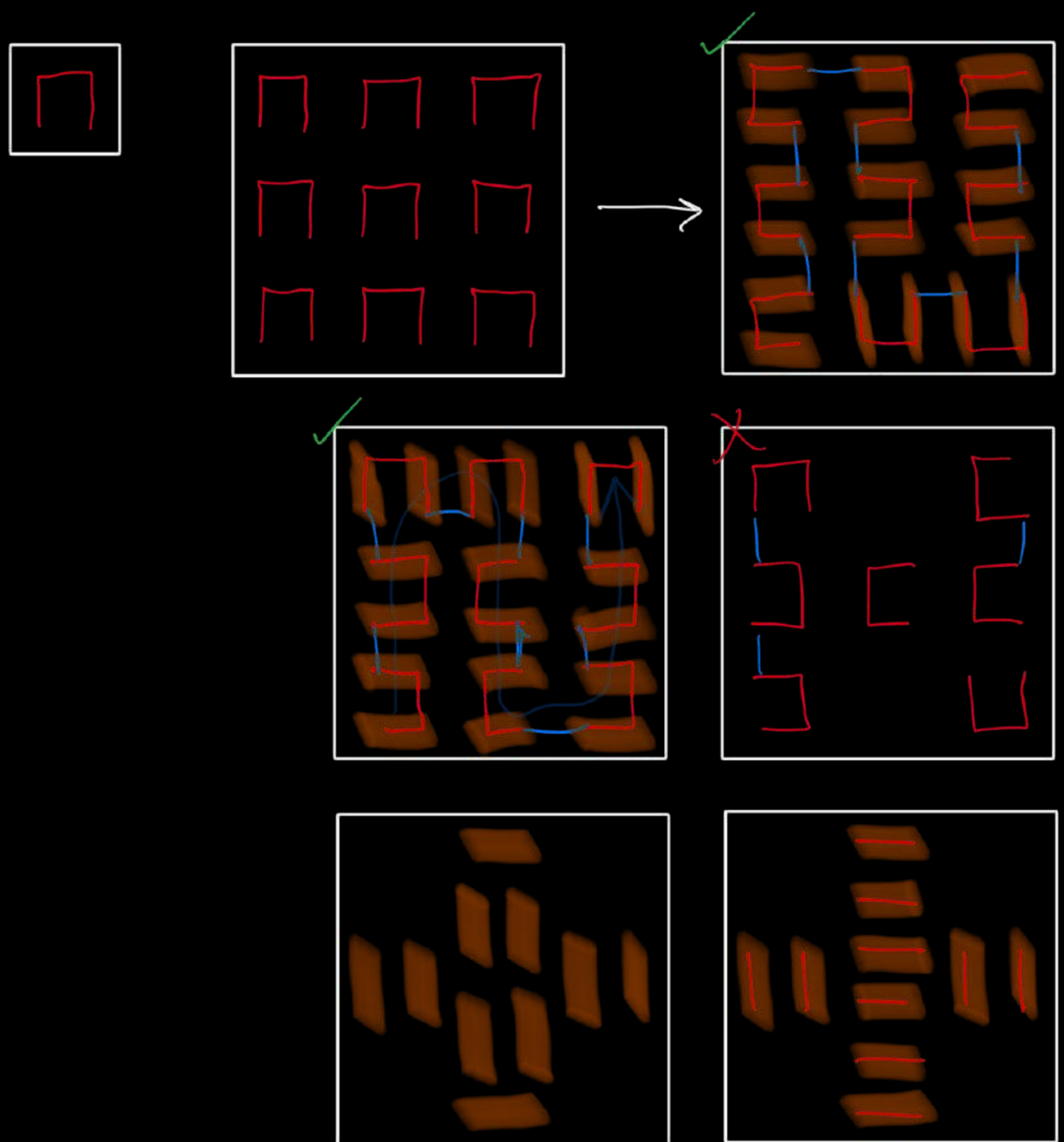
Can I find a fundamental unit?

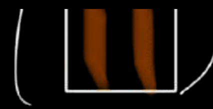


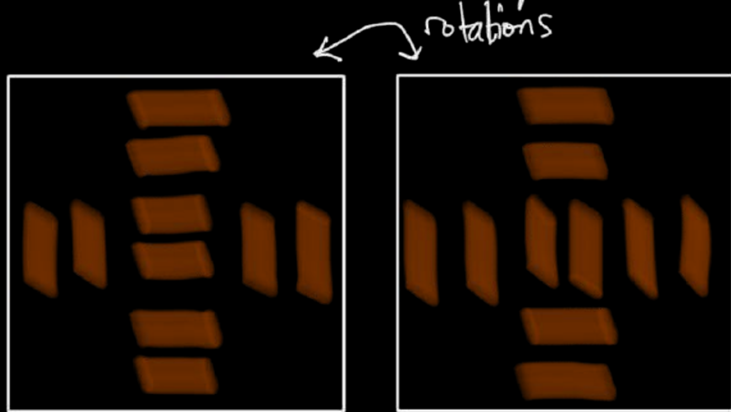









- Let me try a Hilbert style approach, but going from 2x2 to 6x6:



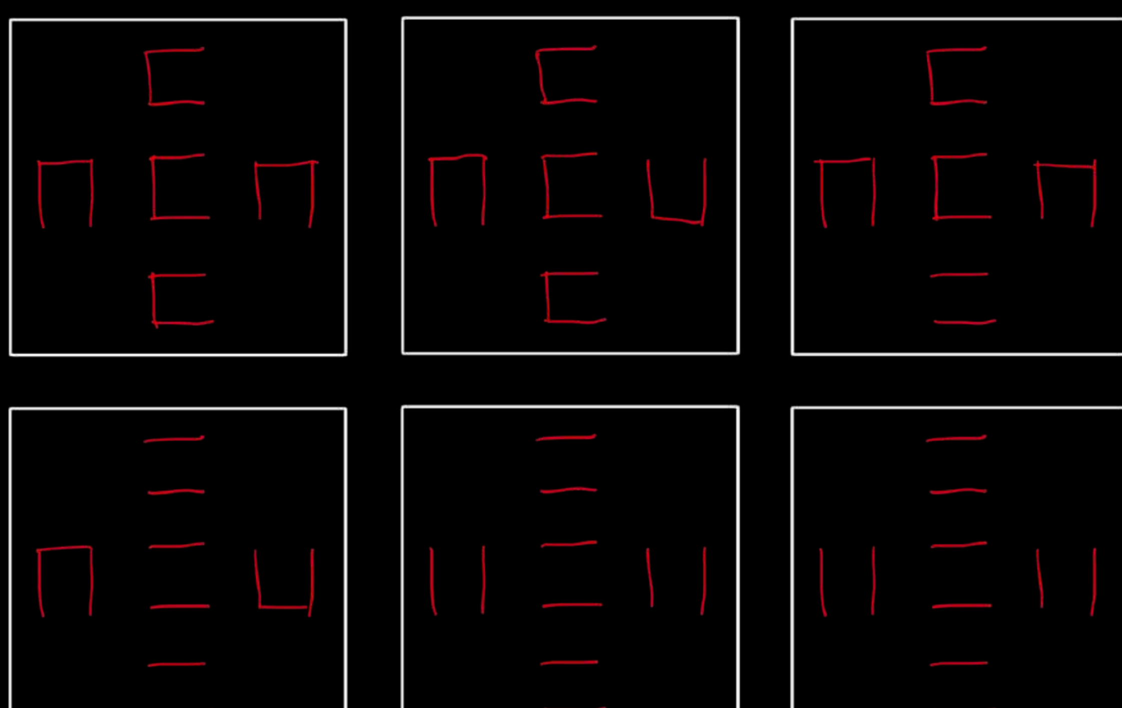
With pairs of two dominoes ()
there are only these options:

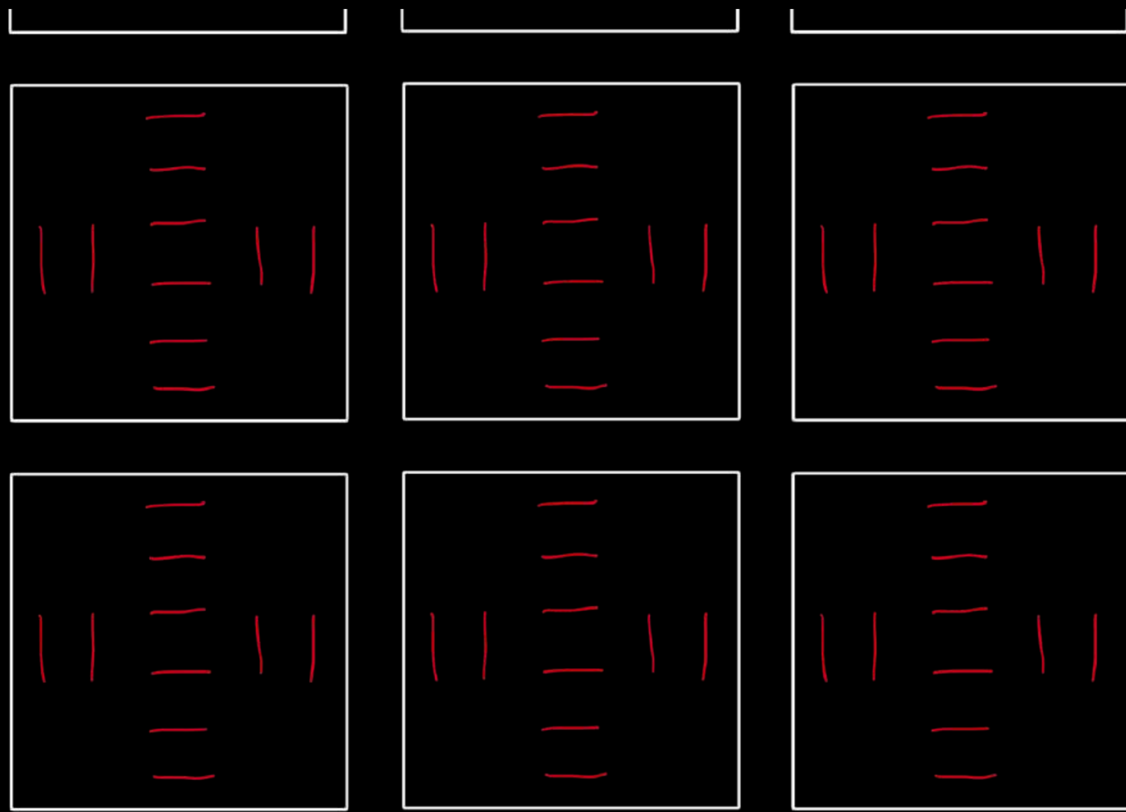


So there's actually only one!!

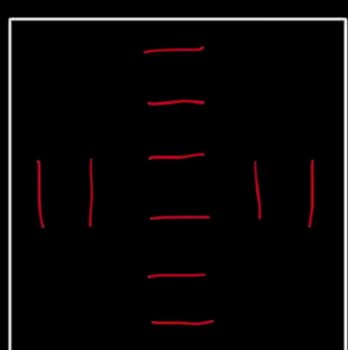
This means these are the options for configs of Hilbert "u"s:
(up to rotational symmetry)

hmm there are a lot...

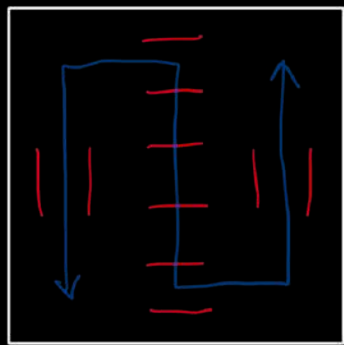
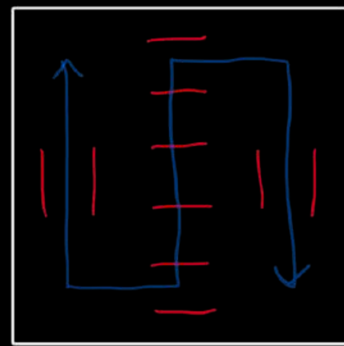
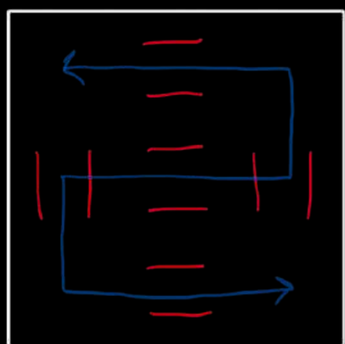
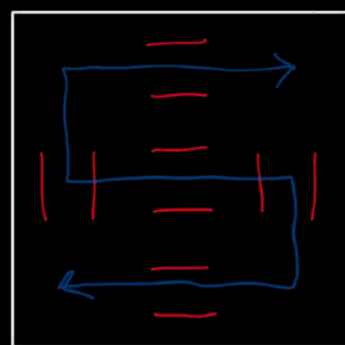
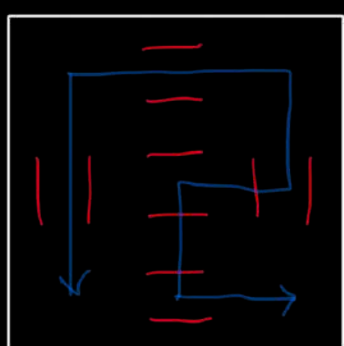
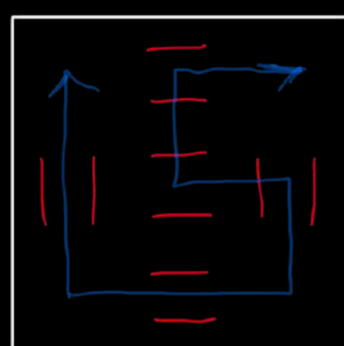
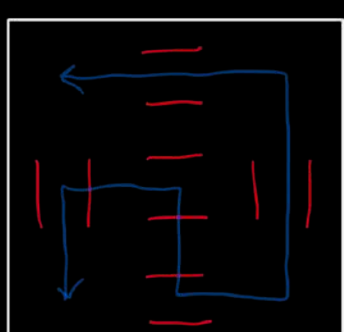
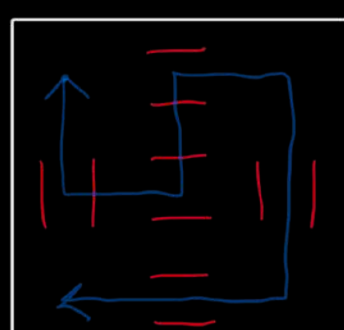


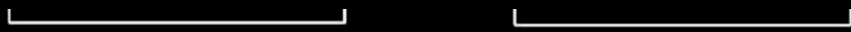


Let me take a step back and start with just this:



Here are the directional paths "through" this, up to rotational symmetry:

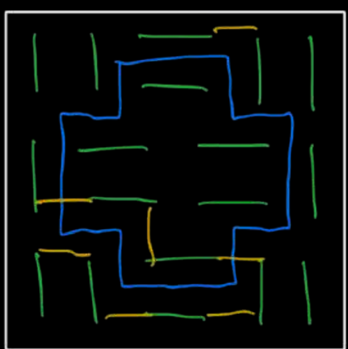









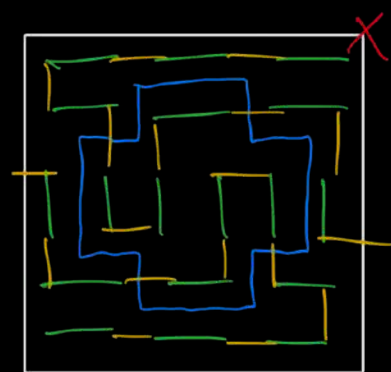


Oh no! This is basically a proof of impossibility, since

doesn't work and every path has one of those.

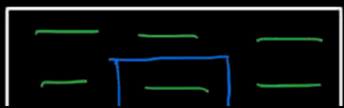
What if I make the fundamental unit a 6x6 with a dot and scale up x3 from there?



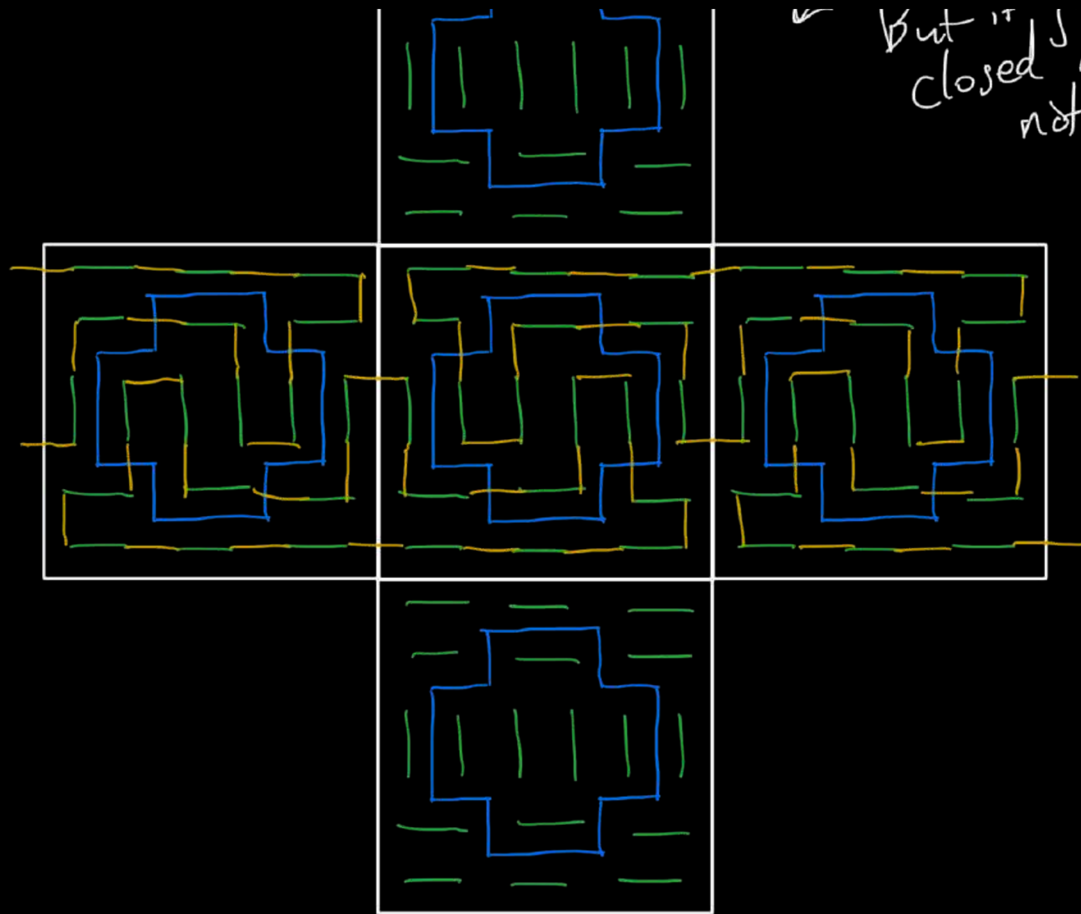


not too terrible — disjoint curve

Can I connect copies of these together?

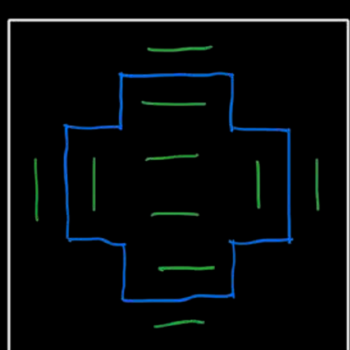


horizontally, yes. [illegible] just makes

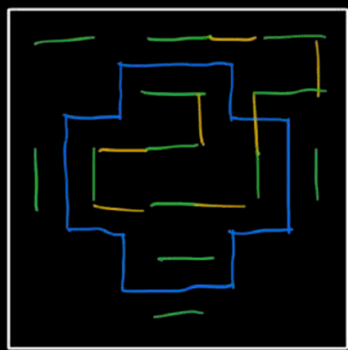


← But it's ... closed loops... not that interesting

I need to find a path through a square with a dot... let me start with that singular arrangement of domino square pairs...

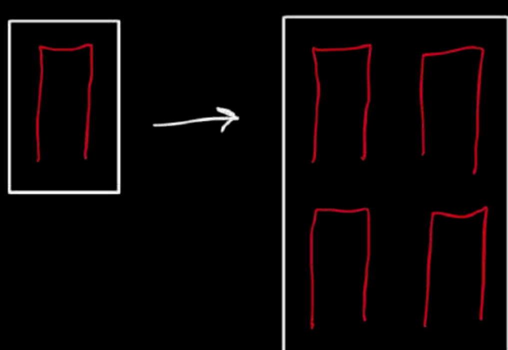


Can I find a path through this?



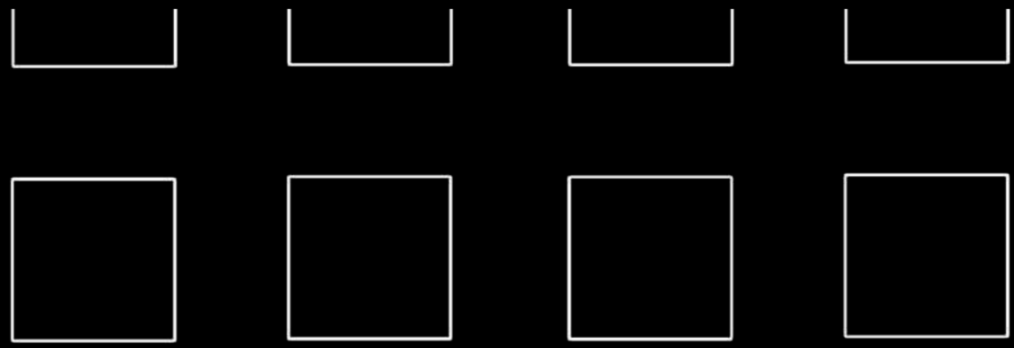
Starting to think this just isn't possible

What if I use a modified, rectangular Hilbert curve and scale from 2x3 to 4x6?



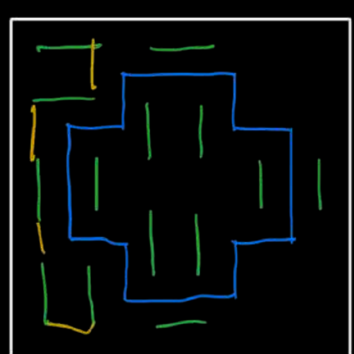
hmm can't rotate

What if I start with 3x3?



No! Can't tile 9 squares with dominoes, dummy

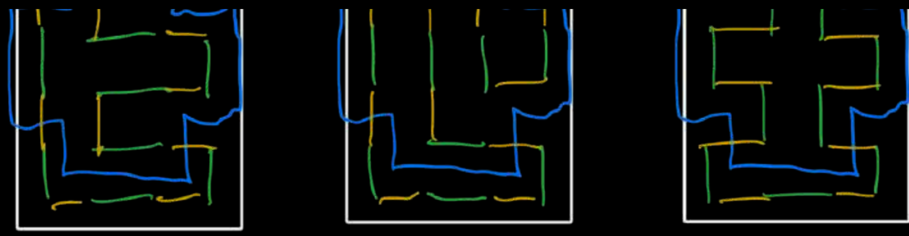
Can I find a path through an arrangement _other_ than the singular domino square pair one?

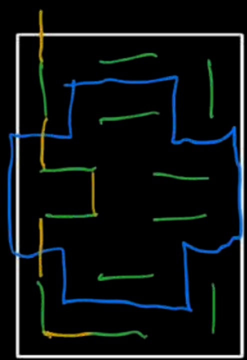


This also seems impossible...

oh wait.

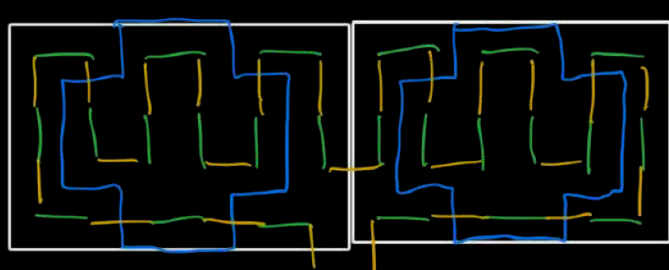
I can exit ~~many~~ two diff ways

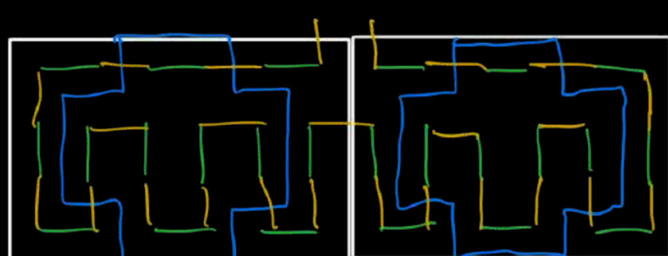


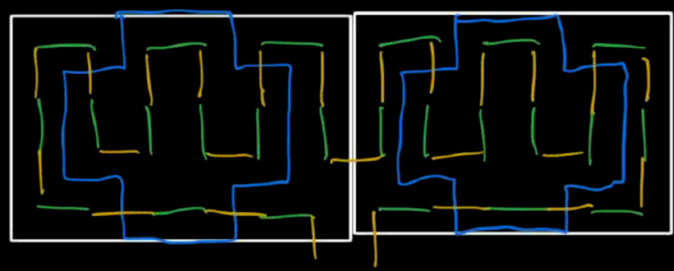


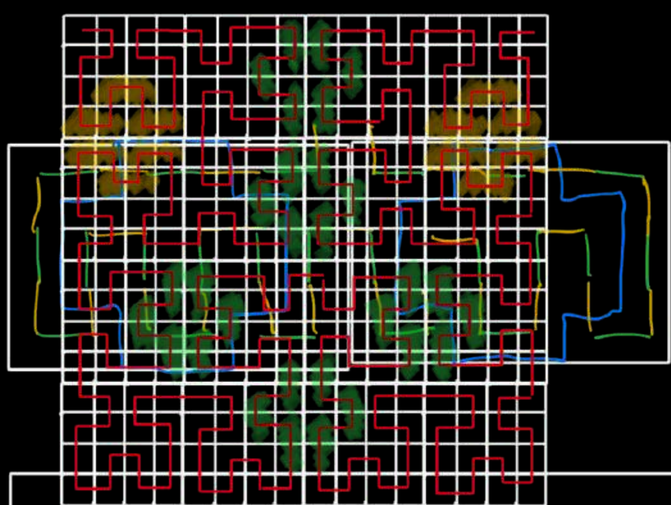
because start + end on same side, will tile into closed loops :(

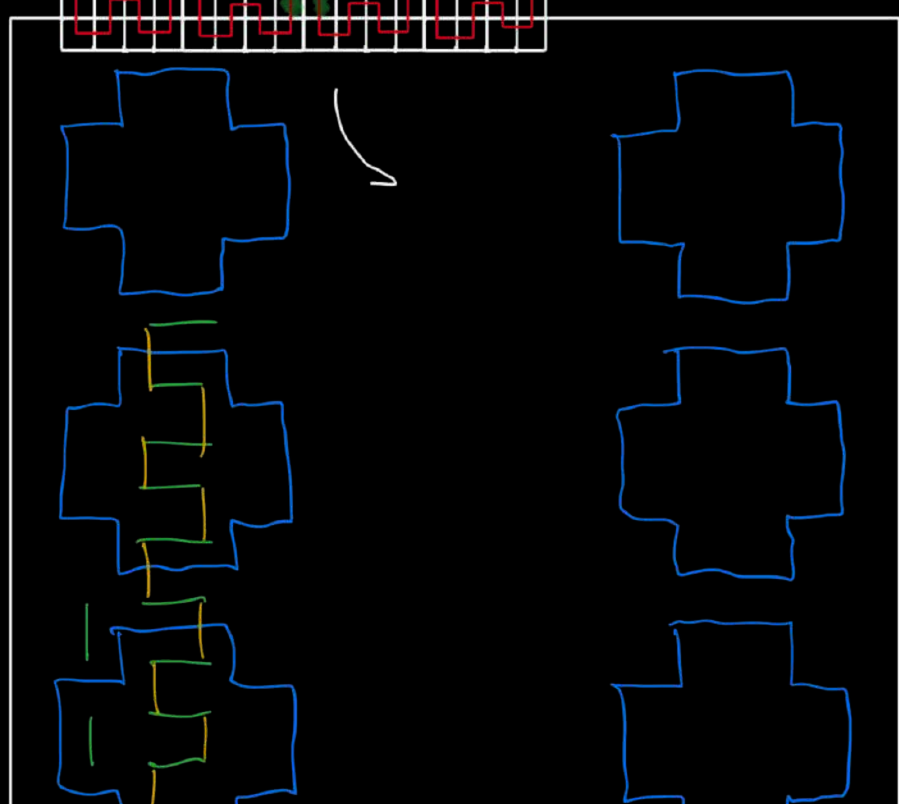
But... maybe I can use this for the dots only and deliberately <u>not</u> connect them...







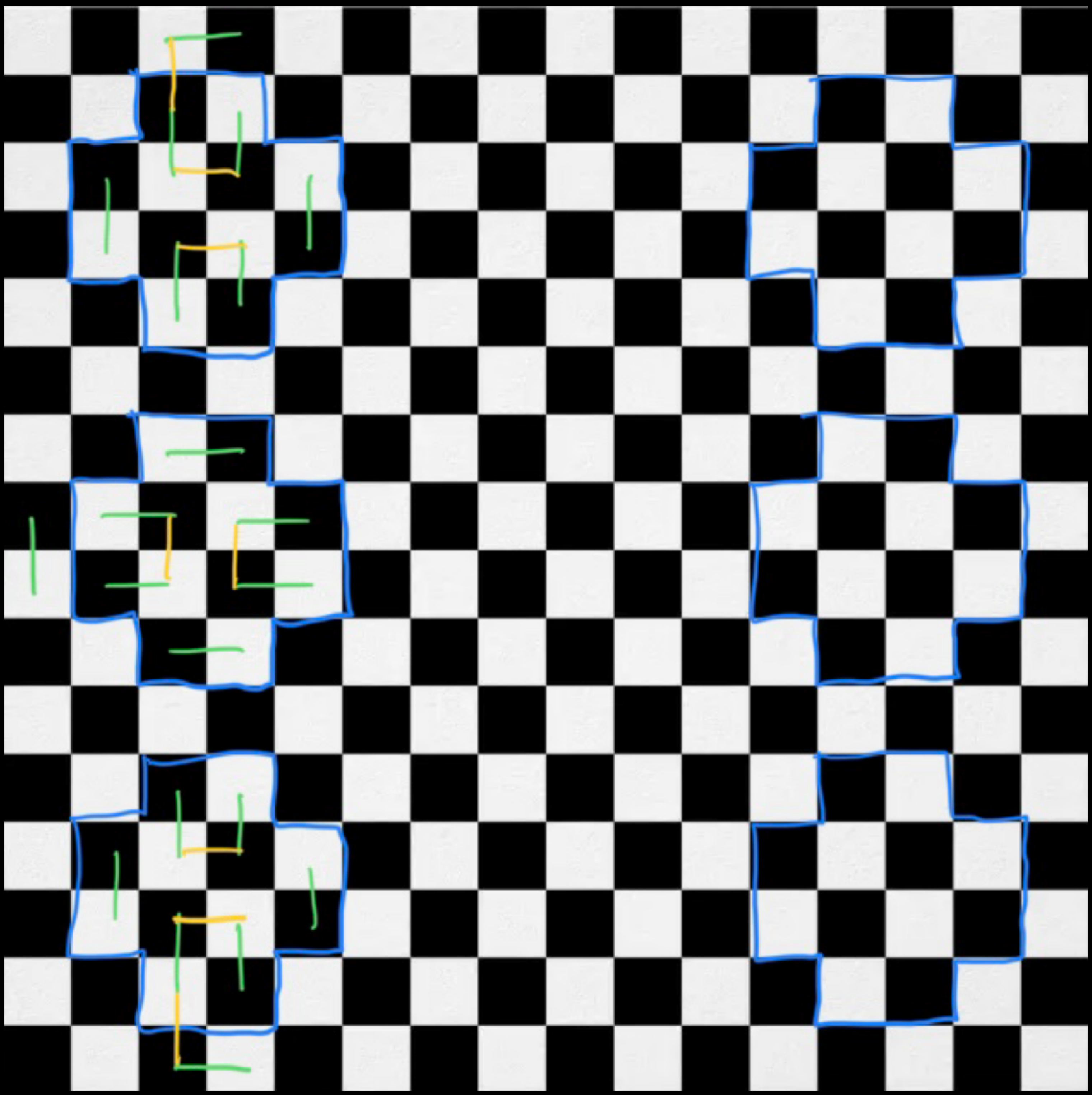




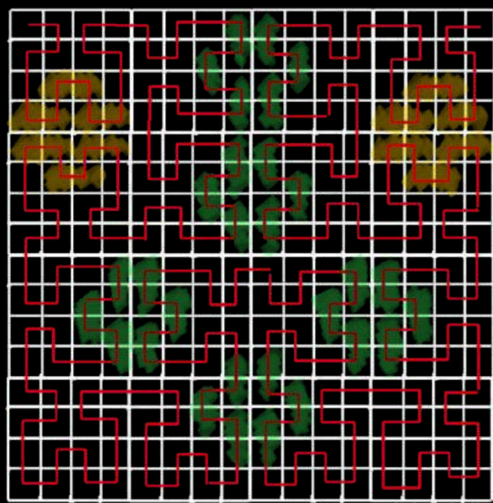
http://gambiter.com/domino/western/math.html

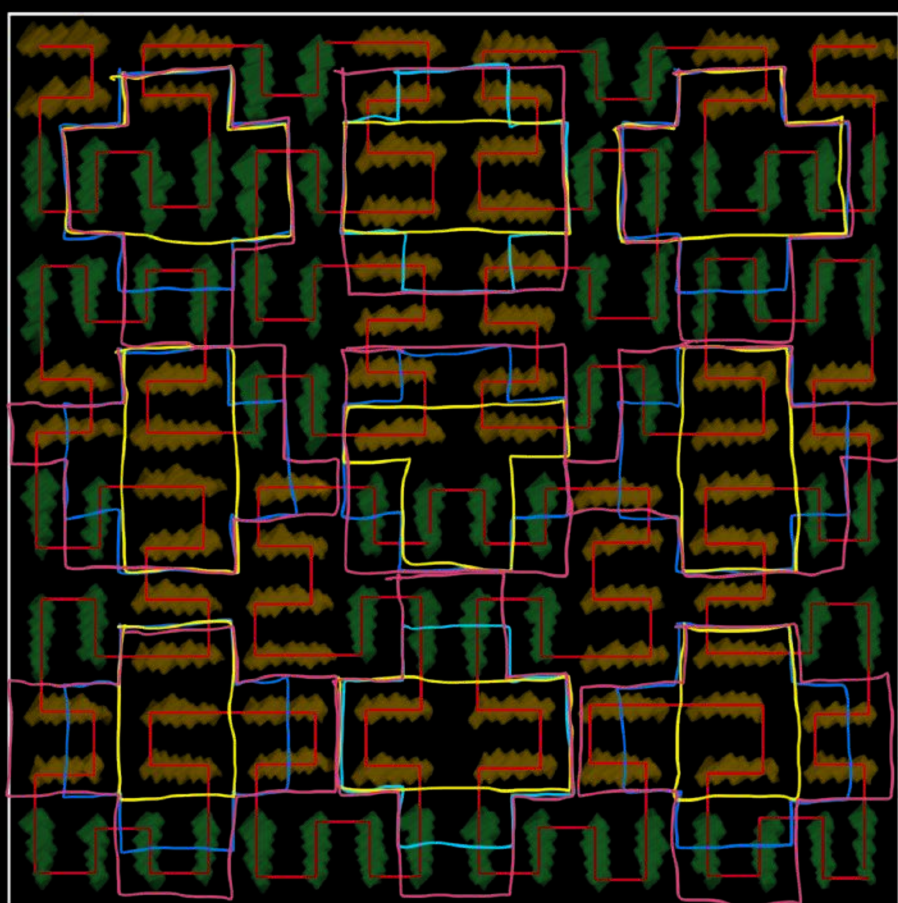
↑ a useful reference

Maybe I can use results about single and circular domino trains, and about tiling checkerboards...

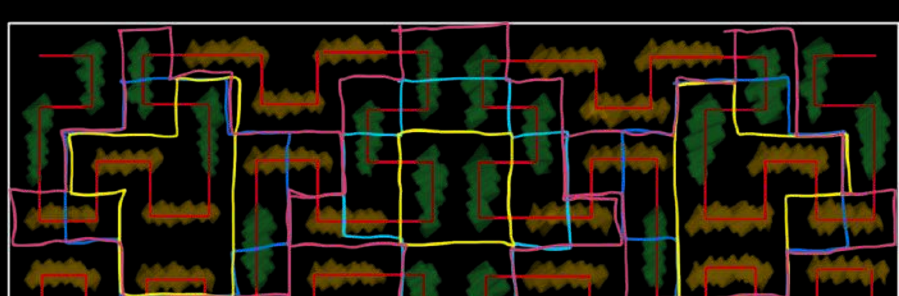


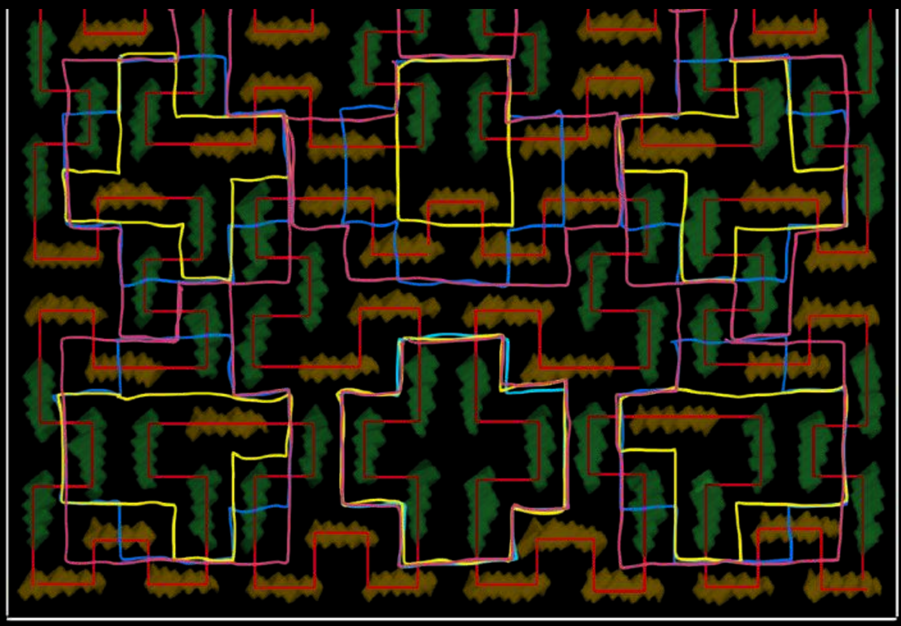
- CHH Dominoes
  - found on Amazon
  - asked them directly, but they don't sell wholesale
  - they'll get back to me with info on where else I might be able to find them





— : dots
— : dots in other orientation
— : staying "inside" dots
— : going "outside"

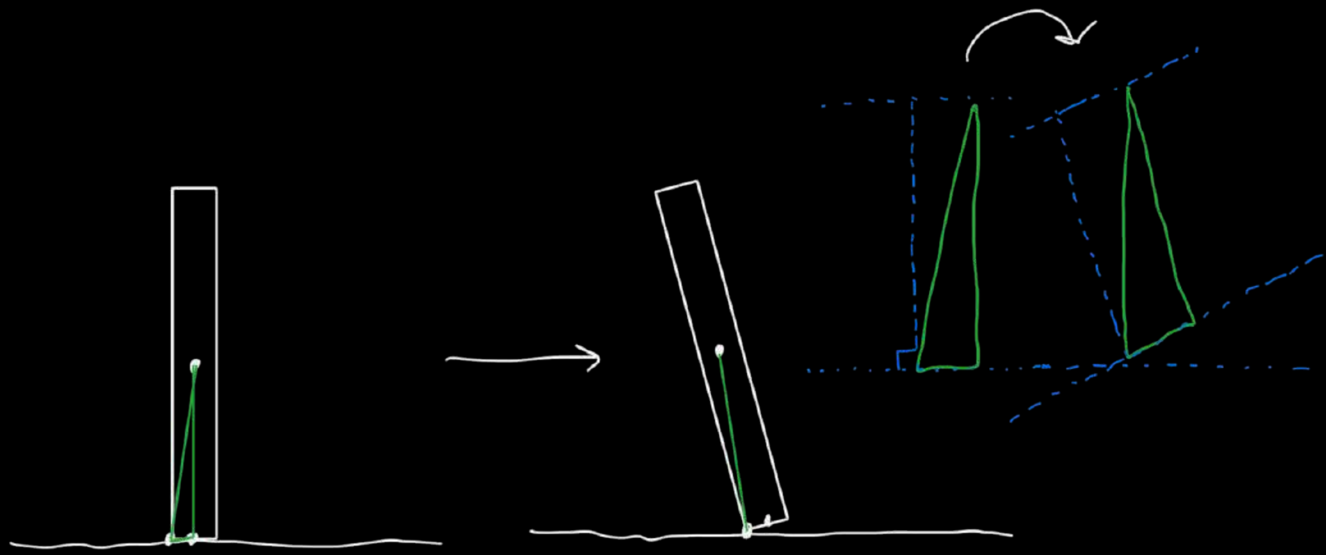


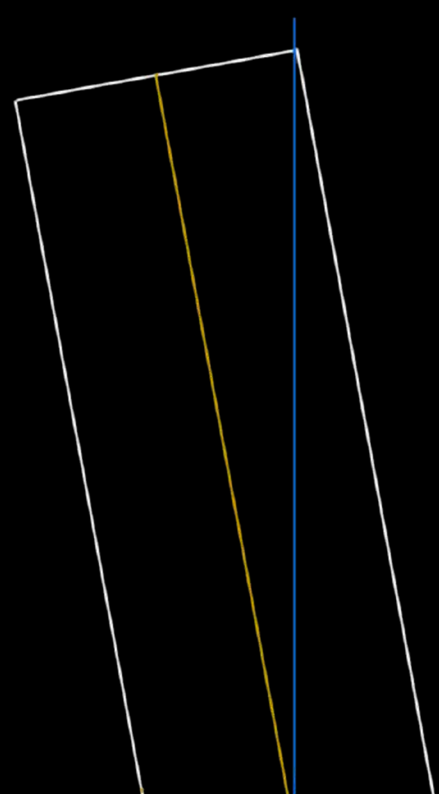


CHH standard domino dimensions: 1.92" x 0.95" x 0.37"

- Center of mass at: (0.96″, 0.475″, 0.185″)
- Triangle from middle of lower edge to middle of lower side to center of mass has side lengths: 0.96″, 0.185″, 0.978″
- So, angle domino has to tip to be perfectly in balance is: sin^{-1}(0.185/0.978) = 10.94° = 0.191 rad
- My (small scale) domino will be 10.08″ x 20.79″ x 2.52″, so displacement (arc) at the top will be: 10.94/360 * 2pi(20.94″) = 4.00″
  - But that's not from vertical.
- IMPORTANT: The above is wrong because my domino of dice doesn't have the same aspect ratio!
- REDO, for my domino of dice:
  - Dimensions: 10.08″ x 20.79″ x 2.52″
  - Center of mass at: (5.04″, 10.40″, 1.26″)
  - Triangle from middle of lower edge to middle of lower side to center of mass has side lengths: 10.40″, 1.26″, 10.48″
  - So, angle domino has to tip to be perfectly in balance is: sin^{-1}(1.26/10.48) = 6.91° =

0.121 rad
  - Displacement (arc) at the top will be: 6.91/360 * 2pi(20.94″) = 2.53″
    - But that's not from vertical.
  - Displacement (arc) at the bottom will be: 6.91/360 * 2pi(2.52″) = 0.30″
    - This is from horizontal.
- If I make it 6 dice thick instead of four, to more closely match the CHH domino dimensions:
  - Dimensions: 10.08″ x 20.79″ x 3.78″
  - Center of mass at: (5.04″, 10.40″, 1.89″)
  - Triangle from middle of lower edge to middle of lower side to center of mass has side lengths: 10.40″, 1.89″, 10.57″
  - So, angle domino has to tip to be perfectly in balance is: sin^{-1}(1.89/10.57) = 10.3° = 0.180 rad
  - Displacement (arc) at the top will be: 10.3/360 * 2pi(21.13″) = 3.80″
    - But that's not from vertical.
  - Displacement (arc) at the bottom will be: 10.3/360 * 2pi(3.78″) = 0.69″
    - This is from horizontal.





https://www.pagat.com/domino/math.html

Helpful reference for math of dominoes, especially trains and circular trains

From above :

- Standard domino set:

28 [
[0,0]
[0,1], [1,1]
[0,2], [1,2], [2,2]
[0,3], [1,3], [2,3], [3,3]
[0,4], [1,4], [2,4], [3,4], [4,4]
[0,5], [1,5], [2,5], [3,5], [4,5], [5,5]
[0,6], [1,6], [2,6], [3,6], [4,6], [5,6], [6,6]
]

21 (from [1,1] through [6,6])

What I want to know

1. Is every sequence a train using all dominoes?
2. Is every sequence a circular train
3. How many sequences are possible?

Starting with #3:

- Suppose you have a set of [m, m] dominoes, so the tiles go from (0,0) through (m, m)

[0,0] through [m,m]

- Write (n_{ia}, n_{ib}) for one tile, where $i = 1, \ldots, M$ is an index for the domino

[0,0] set has 1 domino

[1,1] set has 3

[2,2] set has 6

[3,3] set has 10

$$\# \text{ dominoes} = \binom{m+1}{2} = \frac{(m+1)!}{2!([m+1]-2)!} \equiv M$$

(because dominoes are zero-indexed)

- A train (i.e. a sequence using all tiles) is

$$(n_{1a}, n_{1b}), (n_{2a}, n_{2b}), \ldots, (n_{Ma}, n_{Mb})$$

where $n_{1b} = n_{2a}$, $n_{2b} = n_{3a}$, …, and in general $n_{ib} = n_{(i+1)a}$ for $i = 1, \ldots, M-1$

- Let me start with concrete cases:

[0, 2]:

$[2,2]$

$M=6$

$n_{ia}=n_{ib}=0,1,2$

Suppose $n_{1a}=j$ and $n_{1b}=k$. Then,

$n_{2a}=k$

Notate: $0, 1, 2, \ldots, m$ (arrows under each; j_0 under 0)

- Ugh, my notation SUCKS!

Let's try from scratch...

Set: $[N, N]$ # in Set: $\binom{N+1}{2} = \frac{(N+1)!}{2![(N+1)-2]!} = [$

Tile: $(n_a, n_b)_i$ $n_a=n_b=0,\ldots,N$ $i=1,\ldots,D$

Digits: $d_0,\ldots,d_N \longleftrightarrow 0,\ldots,N$

Tiles with Digit: $d_k \rightarrow (N+1)+1 = N+2$

(b/c zero-indexed → $(N+1)$; b/c one $(d_k, d_k$ → $+1$)

Hmm let's see if the above helps at all...

- Start with any tile.
- There are N+1 options for the one that follows it. That's because there are N+2 tiles with the digit in the second position on that first tile, so minus the first tile there are N+1 remaining
- So, for the first two slots, the number of possibilities is:

$$D \times (N+1) = \frac{(N+1)!}{2![(N+1)-2]!} \times (N+1)$$

- For the third slot, it's trickier. There are either N+1 or N options, depending on whether the first tile has the digit in the second position on the second tile.

Not sure this is worthwhile...

For construction:

Domino of Dice

- Each dice is 0.63"
- Domino is 33 × 16 × 6 dice
- Interior foam is:
  - 31 × 14 × 4 dice
  - 19.53 × 8.82 × 2.52"

Dice of Dominoes

- From measuring w/ the real dominoes:
  - Sides should be 15.625 × 15.625"

Gregory-Leibniz series:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \cdots$$

 $\longleftrightarrow \frac{1}{11}$  $\longleftrightarrow \frac{2}{1}$, etc

Dominoes are base 7 (digits 0 through 6).

base 10: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...
base 7: 0, 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, ...

A number in base x mod x is the last digit of that number. Eg. 13 in base 10 mod 10 is 3, and 13 in base 7 mod 7 is 3. [Note that here I mean "13 in base 7" as the number, not "13" (which we usually implicitly assume is in base 10) in base 7, which would be "16" (in base 7)].

So, let's take this series:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

and make it expressible in dominoes

First, note we can always write eg.

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \cdots$$

So, by convention, as soon as the denominator exceeds +6 (the highest number on a domino), we'll do the following:

1. Increase the numerator by one and

increase [illegible]
adjust the denominator so the fraction is equivalent. E.g. $\frac{1}{7}$ goes to $\frac{2}{14}$, and $\frac{6}{54}$ goes to $\frac{7}{63}$.

2. Express both the numerator and denominator in base 7 mod 7. E.g. $\frac{2}{14}$ goes to $\frac{2}{20}$ and then $\frac{2}{0}$, and $\frac{7}{63}$ goes to $\frac{10}{120}$ and then $\frac{0}{0}$.

In this notation, we have:

$$\frac{\text{"}\pi\text{"}}{4} \rightarrow \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{2}{0} + \frac{2}{4} - \frac{2}{1} + \frac{3}{4} - \frac{3}{3} + \cdots$$

(base 10)

Hmm, don't think this is what I want...
Let me try again: (num and den...)

Suppose I write all the fractions in base 7:

base 10: $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \cdots$

base 7: $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{10} + \frac{1}{12} - \frac{1}{14} + \frac{1}{16} - \frac{1}{21} + \cdots$

And now suppose I "domino encode" this by doing the following:

1. Write the denominators mod 7, thereby keeping only the rightmost digit.
2. Add the cut-off left digit(s) of the denominators to the numerator (eventually writing them mod 7 also when necessary).
3. "Flip" the domino "fraction" when there's a minus sign.

In other words:

$$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{10} + \frac{1}{12} - \frac{1}{14} + \frac{1}{16} - \frac{1}{21} + \cdots$$

$$\downarrow$$

(1) $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{0} + \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{1} + \cdots$

$$\downarrow$$

(2) $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{2}{0} + \frac{2}{2} - \frac{2}{4} + \frac{2}{6} - \frac{3}{1} + \cdots$

$$\downarrow$$

(3) $\boxed{\frac{1}{1}}\ \boxed{\frac{3}{1}}\ \ \boxed{\frac{1}{5}}\ \boxed{\frac{0}{2}}\ \boxed{\frac{2}{2}}\ \boxed{\frac{4}{2}}\ \ \boxed{\frac{2}{6}}\ \boxed{\frac{1}{3}}\ \cdots$

(I can of course also start with 4 in the numerator so the sum equals π.)

Because the sum is conditionally convergent, I can rearrange the values to sum to <u>any</u> real number (including $\pm\infty$!) — see the Riemann series theorem. Equivalently, I can rearrange any finite number of "dominoes", knowing that the rest could be arranged so that the sum still equals π. Thus, I should be able to rearrange the dominoes in a train, but I should check some things first:

- Do all digits occur with equal frequency? From the result of (2) above:
  - The denominator goes 1, 3, 5, 0, 2, 4, 6, 1, 3, 5, ... so yes there.
  - The numerator goes
    1, 1, 1,

2, 2, 2, 2,
3, 3, 3,
4, 4, 4, 4,
5, 5, 5,
6, 6, 6, 6,
0, 0, 0,
1, 1, 1, 1,
2, 2, 2,
3, 3, 3, 3,
4, 4, 4,
5, 5, 5, 5,
6, 6, 6,
0, 0, 0, 0,
1, 1, 1,
...

so yes there too!

- Do all dominoes occur with equal frequency?
  From the result of (2) above:

$\frac{1}{1}, \frac{1}{3}, \frac{1}{5},$
$\frac{2}{0}, \frac{2}{2}, \frac{2}{4}, \frac{2}{6},$
$\frac{3}{1}, \frac{3}{3}, \frac{3}{5},$
$\frac{4}{0}, \frac{4}{2}, \frac{4}{4}, \frac{4}{6},$
$\frac{5}{1}, \frac{5}{3}, \frac{5}{5},$
$\frac{6}{0}, \frac{6}{2}, \frac{6}{4}, \frac{6}{6},$
$\frac{0}{1}, \frac{0}{3}, \frac{0}{5},$
$\frac{1}{0}, \frac{1}{2}, \frac{1}{4}, \frac{1}{6},$
$\frac{2}{1}, \frac{2}{3}, \frac{2}{5},$
$\frac{3}{0}, \frac{3}{2}, \frac{3}{4}, \frac{3}{6},$
$\frac{4}{1}, \frac{4}{3}, \frac{4}{5},$
$\frac{5}{0}, \frac{5}{2}, \frac{5}{4}, \frac{5}{6},$

$\frac{6}{1}, \frac{6}{3}, \frac{6}{5},$

$\frac{0}{0}, \frac{0}{2}, \frac{0}{4}, \frac{0}{6},$

$\frac{1}{1}, \frac{1}{3}, \frac{1}{5},$

...

So yes wow!

$[0,0]^1$

$[0,1]^2, [1,1]^1$

$[0,2]^2, [1,2]^2, [2,2]^1$

$[0,3]^2, [1,3]^2, [2,3]^2, [3,3]^1$

$[0,4]^2, [1,4]^2, [2,4]^2, [3,4]^2, [4,4]^1$

$[0,5]^2, [1,5]^2, [2,5]^2, [3,5]^2, [4,5]^2, [5,5]^1$

$[0,6]^2, [1,6]^2, [2,6]^2, [3,6]^2, [4,6]^2, [5,6]^2, [6,6]^1$

$\Rightarrow$ Not quite — doubles appear ½ as often

Let's try the John Wallis product:

$$\frac{\pi}{2} = \left(\frac{2}{1}\cdot\frac{2}{3}\right)\cdot\left(\frac{4}{3}\cdot\frac{4}{5}\right)\cdot\left(\frac{6}{5}\cdot\frac{6}{7}\right)\cdot\left(\frac{8}{7}\cdot\frac{8}{9}\right)$$

$$\cdot\left(\frac{10}{9}\cdot\frac{10}{11}\right)\cdot\left(\frac{12}{11}\cdot\frac{12}{13}\right)\cdot\left(\frac{14}{13}\cdot\frac{14}{15}\right)\cdot\left(\frac{16}{15}\cdot\frac{16}{17}\right)$$

$$\cdot\left(\frac{18}{17}\cdot\frac{18}{19}\right)\cdot\left(\frac{20}{19}\cdot\frac{20}{21}\right)\cdot\left(\frac{22}{21}\cdot\frac{22}{23}\right)\cdot\left(\frac{24}{23}\cdot\frac{24}{25}\right)\cdots$$

$$= \prod_{n=1}^{\infty}\frac{4n^2}{4n^2-1} = \prod_{n=1}^{\infty}\left(\frac{2n}{2n-1}\cdot\frac{2n}{2n+1}\right)$$

Writing the fractions mod 7:

repeating unit:

$$\left(\frac{2}{1}\cdot\frac{2}{3}\right)\left(\frac{4}{3}\cdot\frac{4}{5}\right)\left(\frac{6}{5}\cdot\frac{6}{0}\right)\cdot\left(\frac{1}{0}\cdot\frac{1}{2}\right)$$

$$\cdot\left(\frac{3}{2}\cdot\frac{3}{4}\right)\cdot\left(\frac{5}{4}\cdot\frac{5}{6}\right)\cdot\left(\frac{0}{6}\cdot\frac{0}{1}\right)\left(\frac{2}{1}\cdot\frac{2}{3}\right)$$

$$\cdot\left(\frac{4}{3}\cdot\frac{4}{5}\right)\cdot\left(\frac{6}{5}\cdot\frac{6}{0}\right)\left(\frac{1}{0}\cdot\frac{1}{2}\right)\left(\frac{3}{2}\cdot\frac{3}{4}\right)\cdots$$

What I like about this is it naturally forms a train!

[1,2] [2,3] [3,4] [4,5] [5,6] [6,0] [0,1] ...

Oh wait I didn't start by writing in base 7.

Maybe I shouldn't care? Yeah, I think it's good like this.

But the problem is, there are no doubles at all...

Let me try base 7:

$$\left(\frac{2}{1}\cdot\frac{2}{3}\right)\cdot\left(\frac{4}{3}\cdot\frac{4}{5}\right)\cdot\left(\frac{6}{5}\cdot\frac{6}{10}\right)\cdot\left(\frac{11}{10}\cdot\frac{11}{12}\right)$$
$$\cdot\left(\frac{13}{12}\cdot\frac{13}{14}\right)\cdot\left(\frac{15}{14}\cdot\frac{15}{16}\right)\cdot\left(\frac{20}{16}\cdot\frac{20}{21}\right)\cdot\left(\frac{22}{21}\cdot\frac{22}{23}\right)\cdots$$

What if I take the mod 7 result above and just added 1 to the numerators for every subsequent repeating unit?!

pretty cool, there's a kind of "mirror" here

— : already counted

$$\left(\frac{2}{1}\cdot\frac{2}{3}\right)\left(\frac{4}{3}\cdot\frac{4}{5}\right)\left(\frac{6}{5}\cdot\frac{6}{0}\right)\cdot\left(\frac{1}{0}\cdot\frac{1}{2}\right)$$
$$\cdot\left(\frac{3}{2}\cdot\frac{3}{4}\right)\cdot\left(\frac{5}{4}\cdot\frac{5}{6}\right)\cdot\left(\frac{0}{6}\cdot\frac{0}{1}\right)$$

$$\cdot\left(\frac{3}{1}\cdot\frac{3}{3}\right)\cdot\left(\frac{5}{3}\cdot\frac{5}{5}\right)\cdot\left(\frac{0}{5}\cdot\frac{0}{0}\right)\cdot\left(\frac{2}{0}\cdot\frac{2}{2}\right)$$
$$\cdot\left(\frac{4}{2}\cdot\frac{4}{4}\right)\cdot\left(\frac{6}{4}\cdot\frac{6}{6}\right)\cdot\left(\frac{1}{6}\cdot\frac{1}{1}\right)$$

$$\cdot\left(\frac{4}{1}\cdot\frac{4}{3}\right)\cdot\left(\frac{6}{3}\cdot\frac{6}{5}\right)\cdot\left(\frac{1}{5}\cdot\frac{1}{0}\right)\cdot\left(\frac{3}{0}\cdot\frac{3}{2}\right)$$
$$\cdot\left(\frac{5}{2}\cdot\frac{5}{4}\right)\left(\frac{0}{4}\cdot\frac{0}{6}\right)\cdot\left(\frac{2}{6}\cdot\frac{2}{1}\right)$$

$$\cdot\left(\frac{5}{1}\cdot\frac{5}{3}\right)\left(\frac{0}{3}\cdot\frac{0}{5}\right)\cdot\left(\frac{2}{5}\cdot\frac{2}{0}\right)\cdot\left(\frac{4}{0}\cdot\frac{4}{2}\right)$$
$$\cdot\left(\frac{6}{2}\cdot\frac{6}{4}\right)\cdot\left(\frac{1}{4}\cdot\frac{1}{6}\right)\cdot\left(\frac{3}{6}\cdot\frac{3}{1}\right)$$

$$\cdot\left(\frac{6}{1}\cdot\frac{6}{3}\right)\cdot\left(\frac{1}{3}\cdot\frac{1}{5}\right)\cdot\left(\frac{3}{5}\cdot\frac{3}{0}\right)\cdot\left(\frac{5}{0}\cdot\frac{5}{2}\right)$$
$$\cdot\left(\frac{0}{2}\cdot\frac{0}{4}\right)\cdot\left(\frac{2}{4}\cdot\frac{2}{6}\right)\cdot\left(\frac{4}{6}\cdot\frac{4}{1}\right)$$

$$\left(\frac{0}{1}\cdot\frac{0}{3}\right)\cdot\left(\frac{2}{3}\cdot\frac{2}{5}\right)\cdot\left(\frac{4}{5}\cdot\frac{4}{0}\right)\cdot\left(\frac{6}{0}\cdot\frac{6}{2}\right)$$

$\cdot\left(\frac{\ }{1}\ \frac{\ }{3}\right)\left(\frac{\ }{3}\ \frac{\ }{5}\right)\left(\frac{\ }{5}\ \frac{\ }{0}\right)\left(\frac{\ }{0}\ \frac{\ }{2}\right)$

$\cdot\left(\frac{1}{2}\cdot\frac{1}{4}\right)\cdot\left(\frac{3}{4}\cdot\frac{3}{6}\right)\cdot\left(\frac{5}{6}\ \frac{5}{1}\right)$

$\cdot\left(\frac{1}{1}\cdot\frac{1}{3}\right)\cdot\left(\frac{3}{3}\cdot\frac{3}{5}\right)\cdot\left(\frac{5}{5}\cdot\frac{5}{0}\right)\cdot\left(\frac{0}{0}\cdot\frac{0}{2}\right)$

$\cdot\left(\frac{2}{2}\cdot\frac{2}{4}\right)\cdot\left(\frac{4}{4}\cdot\frac{4}{6}\right)\cdot\left(\frac{6}{6}\cdot\frac{6}{1}\right)$

(then, whole above sequence repeats)

This looks promising!

- Do all digits occur w/ equal frequency? ✓
- Do all dominoes occur w/ equal frequency?

$[0,0]^2$

$[0,1]^4$, $[1,1]^2$

$[0,2]^4$, $[1,2]^4$, $[2,2]^2$

$[0,3]^4$, $[1,3]^4$, $[2,3]^4$, $[3,3]^2$

$[0,4]^4$, $[1,4]^4$, $[2,4]^4$, $[3,4]^4$, $[4,4]^2$

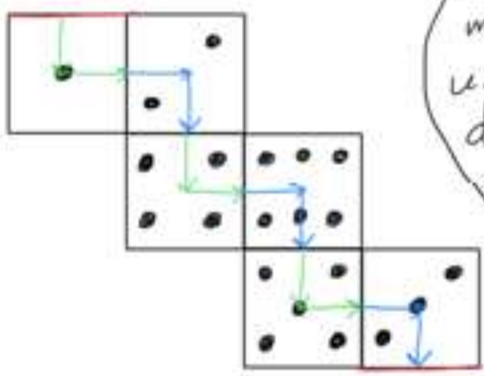
$[0,5]^4$, $[1,5]^4$, $[2,5]^4$, $[3,5]^4$, $[4,5]^4$, $[5,5]^2$

$[0,6]^4$, $[1,6]^4$, $[2,6]^4$, $[3,6]^4$, $[4,6]^4$, $[5,6]^4$, $[6,6]^2$

So like before (with Gregory-Leibniz), not quite but close. The doubles appear ½ as often. But this has a huge advantage over Gregory-Leibniz: it naturally forms a train, with a little "flipping"!

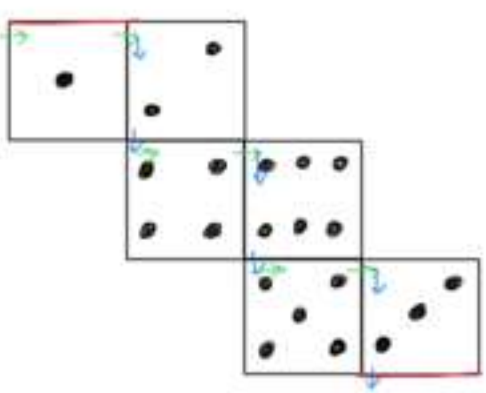
Now I need to sort out what Hilbert-type curve I should use to tile the cube...

- One way to "unfold":



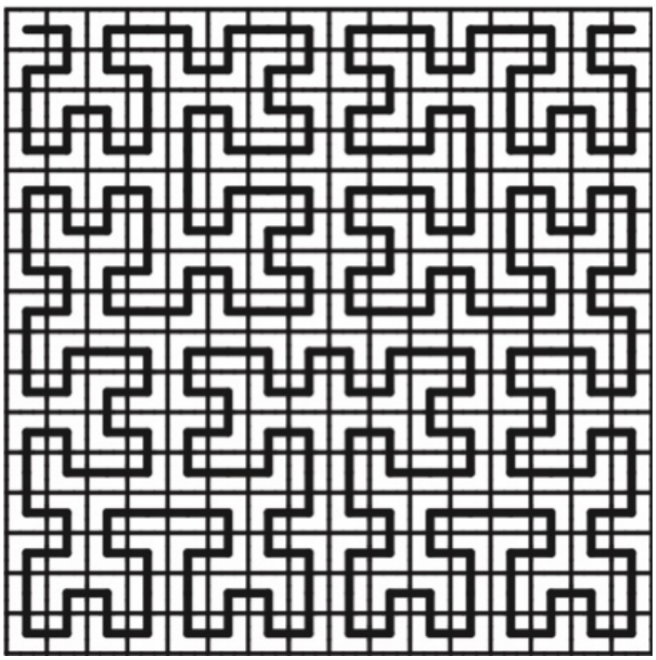
(made sure to use an actual dice to get the dot orientations right)

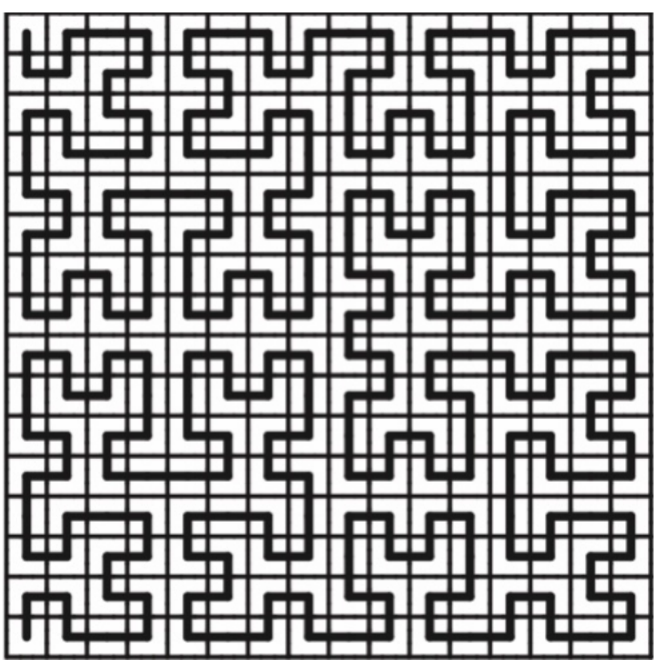
- So, I can do this with two curve orientations (green and blue above), each going from the center to a side's middle
- But, I'd prefer to use the "standard" Hilbert curve and rotations of it, since that's more recognizable. And I think I can do that:



- Green is a standard Hilbert

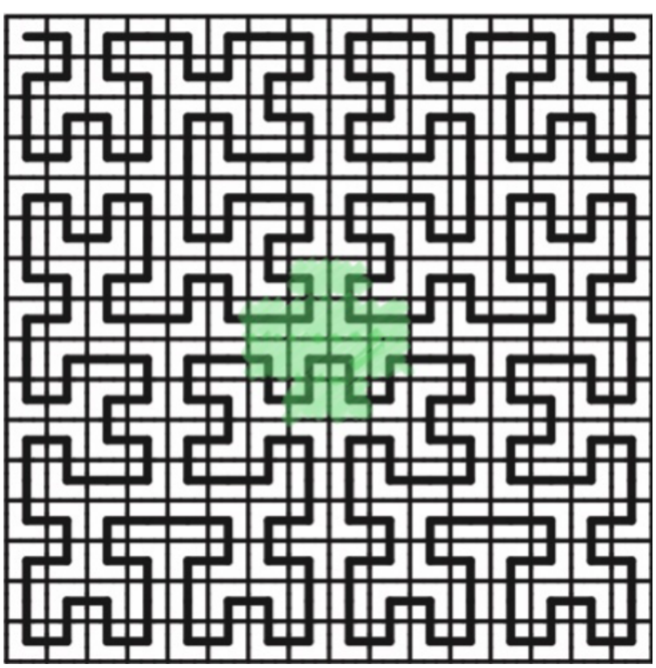
curve starting in the upper left corner and ending in the upper right. Blue is that rotated 90° right.

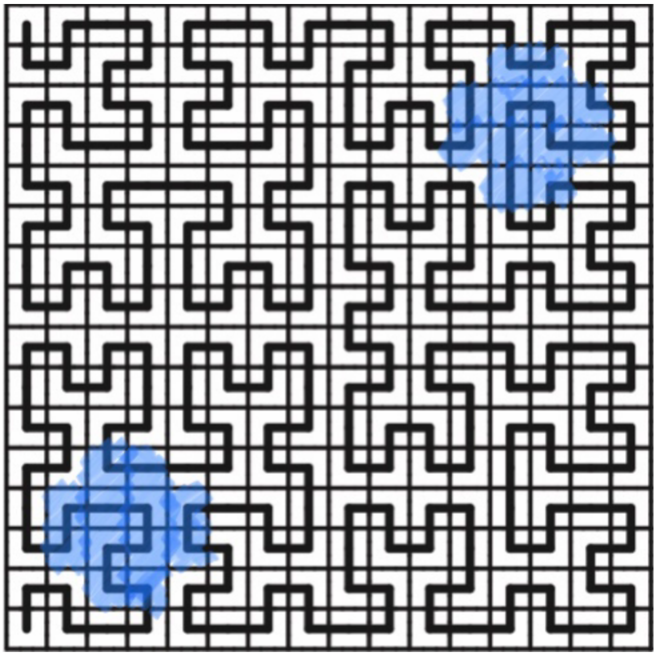


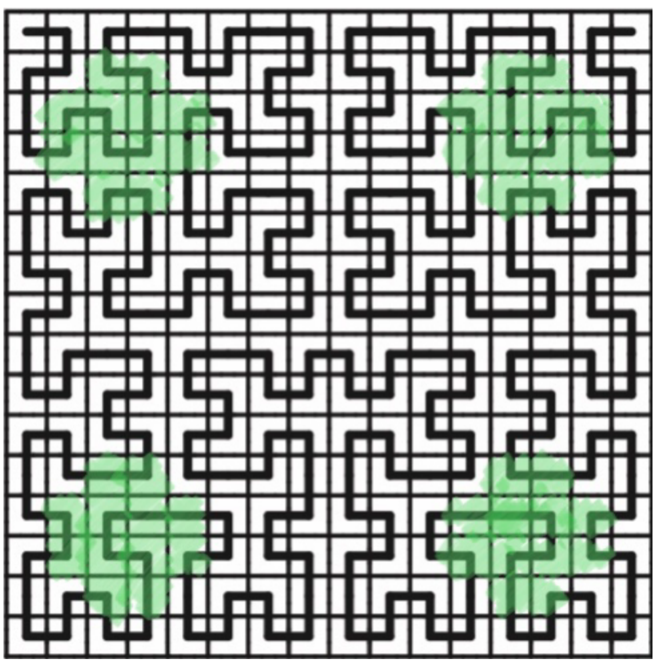


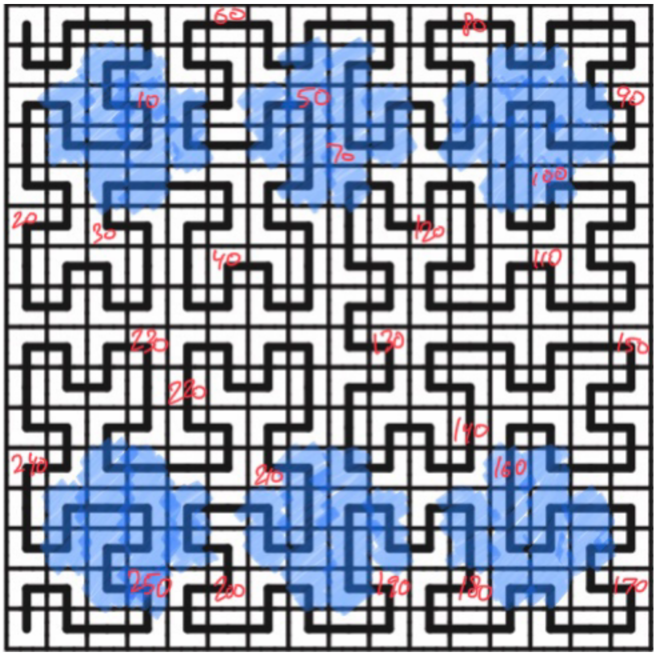
(Green is top. Blue is bottom)

- Now, I need to determine which dominoes I'll use to see if I have enough. Let's start by getting the positions of the dots. 1, 4, and 5 are green type; 2, 6, and 3 are blue type.








60
80
10
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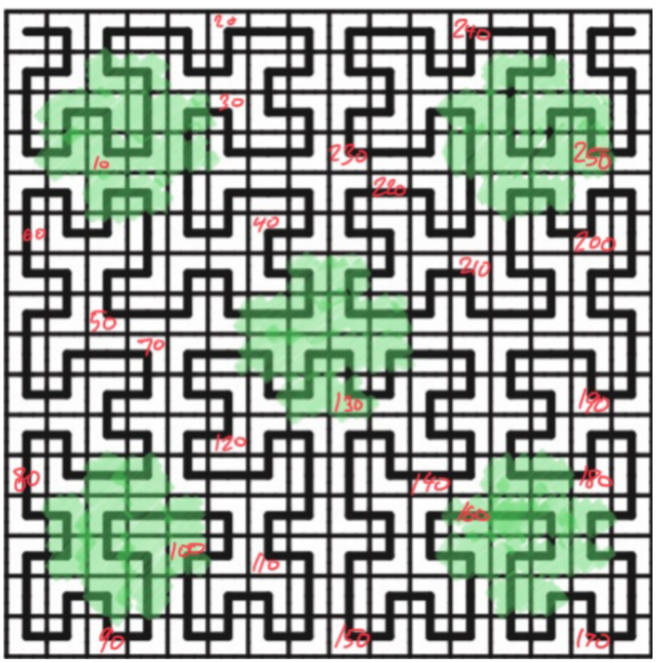
180
170


20
240
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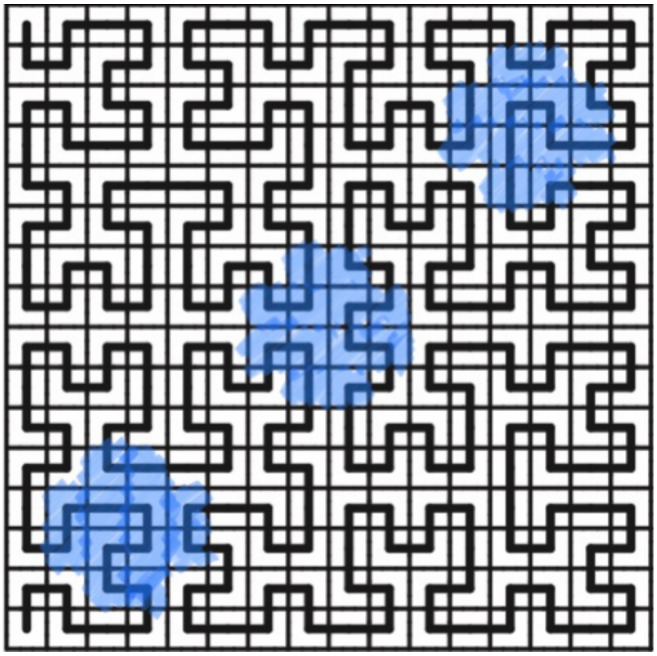
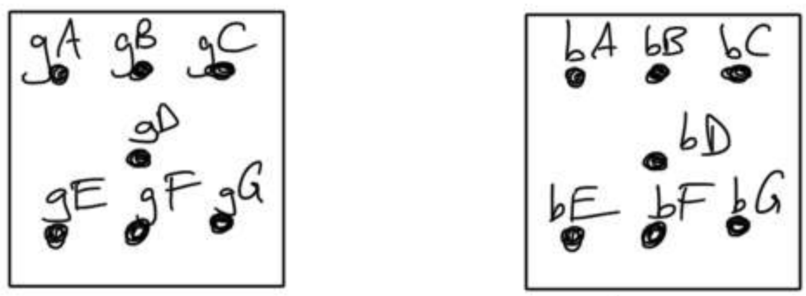
150
170



- I'm going to label the dot positions so I can talk about dots in the same position on multiple faces:



green blue

(gB and gF are never used)

- For type green, label the squares 1, 2, 3, ... starting in the upper left and for type blue, same.

- Let's write out which squares each dot includes:

frequency

2 • gA: 7-14, 31-32, 54-55

0 • gB: (N/A)

2 • gC: 202-203, 225-226, 243-250

2 • gD: 42-44, 125, 127-130, 132, 213-215

2 • gE: 75-76, 82-83, 89, 92-97, 100

0 • gF: (N/A)

2 • gG: 157, 160-165, 168, 174-175, 181-182

1 • hA: Same as aC

1 • bA : same as gE

1 • bB : 48-52, 63, 68-72, 123

3 • bC : same gG

1 • bD : same as gD

3 • bE : same as gA

1 • bF : 134, 185-189, 194, 205-209

1 • bG : same as gE

– The Wallis product is:

$$\prod_{n=1}^{\infty} \frac{4n^2}{4n^2-1} = \prod_{n=1}^{\infty} \left(\frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \right)$$

I'm staying true to the denominators of this, but I'm adding 1 to the numerators for every multiple of 7 in n. (And I'm writing the fractions mod 7.) So, my "domino train" product is:

$$\prod_{n=1}^{\infty} \frac{\left(2n + \frac{\lfloor (n-1)/7 \rfloor}{7}\right) \bmod 7}{(2n-1) \bmod 7} \cdot \frac{\left(2n + \frac{\lfloor (n-1)/7 \rfloor}{7}\right) \bmod}{(2n+1) \bmod 7}$$

where $\left\lfloor \frac{n-1}{7} \right\rfloor$ is the floor where

function (or integer division) of $\frac{n-1}{7}$. For $n-1 = 0, \ldots, 6$, that's 0. For $n-1 = 7, \ldots, 13$, that's 1. Etc.

- Each of these terms can be considered a "domino" though they'll be offset by one since I'm using the (more interesting looking) tiling with half dominoes at the beginning and end

Manually counting from my spreadsheet...

Full black dominoes

— : type 1 tiling
— : type 2 tiling
— : total

$[0,0]^{0}$

$[0,1]^{3}$, $[1,1]^{2}$

$[0,2]^{4}$, $[1,2]^{2}$, $[2,2]^{2}$

$[0,3]^{1}$, $[1,3]^{0}$, $[2,3]^{1}$, $[3,3]^{0}$

$[0,4]^{2}$, $[1,4]^{0}$, $[2,4]^{1}$, $[3,4]^{4}$, $[4,4]^{1}$

$[0,5]^{0}$, $[1,5]^{0}$, $[2,5]^{2}$, $[3,5]^{2}$, $[4,5]^{2}$, $[5,5]^{0}$

$[0,6]^{3}$, $[1,6]^{2}$, $[2,6]^{0}$, $[3,6]^{2}$, $[4,6]^{3}$, $[5,6]^{4}$, $[6,6]^{3}$

Partial black dominoes

$[0,?]^{9}$, $[1,?]^{8}$, $[2,?]^{9}$, $[3,?]^{10}$, $[4,?]^{8}$, $[5,?]^{5}$, $[6,?]^{3}$

Full white dominoes

$[0,0]^{4}$

[0, 1]^7, [1, 1]^0

[0, 2]^4, [1, 2]^4, [2, 2]^3

[0, 3]^4, [1, 3]^7, [2, 3]^9, [3, 3]^2

[0, 4]^2, [1, 4]^10, [2, 4]^5, [3, 4]^10, [4, 4]^3

[0, 5]^8, [1, 5]^8, [2, 5]^3, [3, 5]^6, [4, 5]^6, [5, 5]^6

[0, 6]^9, [1, 6]^5, [2, 6]^5, [3, 6]^5, [4, 6]^7, [5, 6]^6, [6, 6]^3

Partial white dominoes

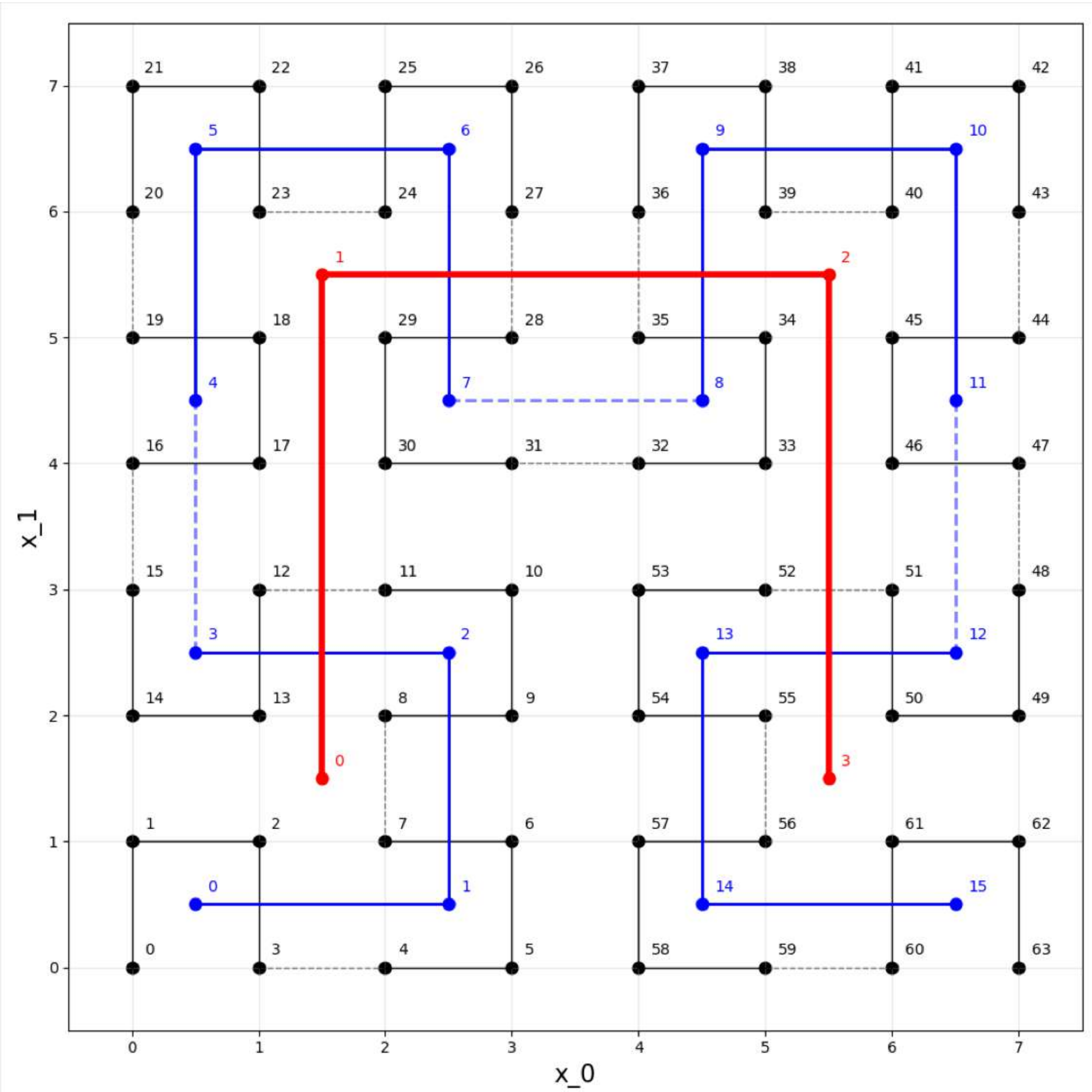
[0, ?]^6, [1, ?]^8, [2, ?]^13, [3, ?]^9, [4, ?]^8, [5, ?]^8, [6, ?]^4

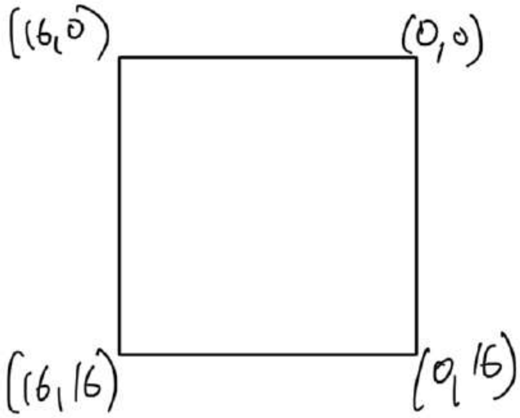
(So crazy the [1, 1] domino never appears in white! I double checked this, and it looks to be right.)

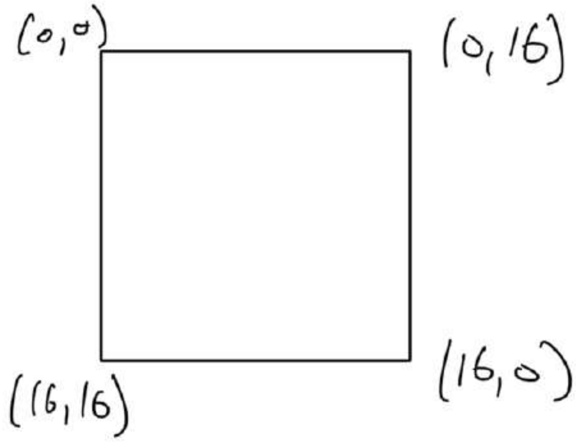
Shoot, I actually need to count

on a per dot basis because they occur with different frequencies

Helpful Python library: https://pypi.org/project/hilbertcurve/

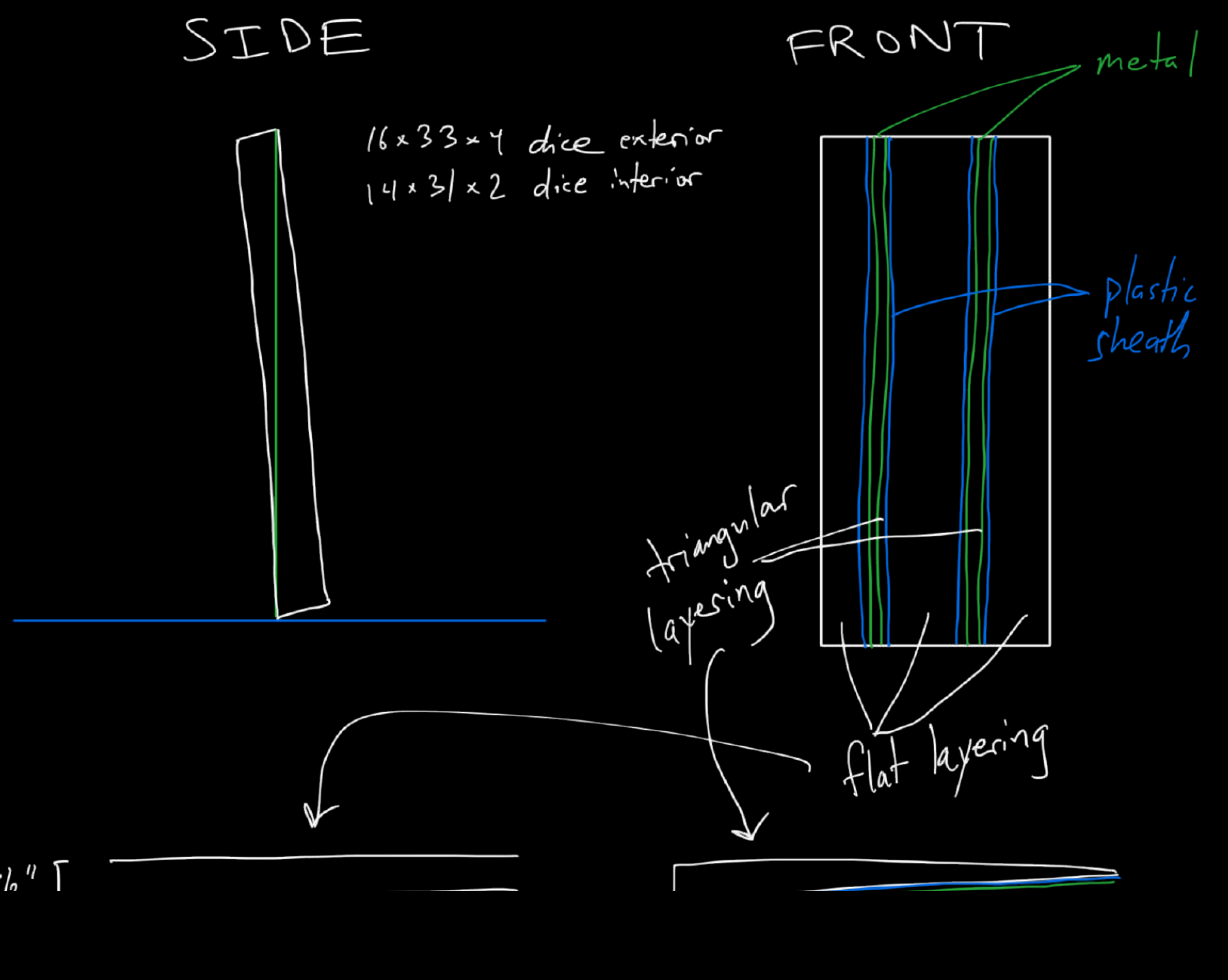



(16,0)
(0,0)
(16,16)
(0,16)

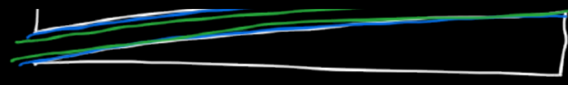

(0,0)
(0,16)
(16,16)
(16,0)

Construction

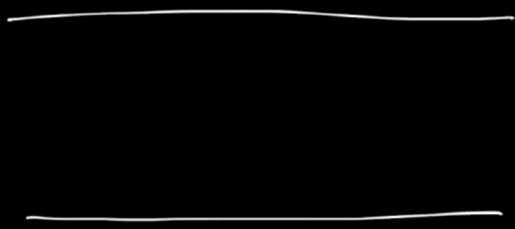
- I have 1/2" white and 3/16" black foam board
- (2 × 1/2") + (3/16") seems to exactly match 2 dice and the plastic wire channel I have (width wise)
- To go 1/2 dice down, I can cut a hole in the 1/2" and add a piece of the 3/16"
- To go a full dice down, I can cut a hole in 2 1/2" pieces and add 2 pieces of 3/16"
- Construction plan:



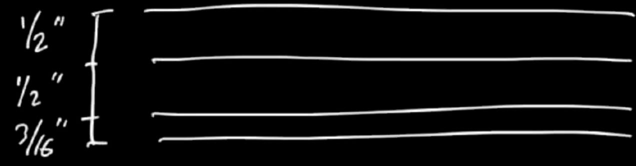
(side)



(side)



(front)



(side)

Measurements on groups of actual dice:

4 across: 2.50"

16 across: 10.05"

33 across: 20.70"

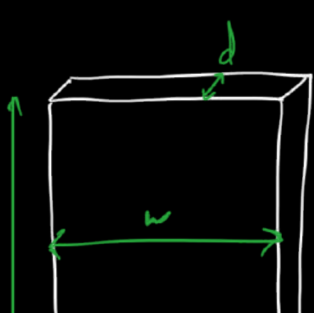
2 across: 1.25"

14 across: 8.80"

31 across: 19.45"

Dimensions of internal structure:

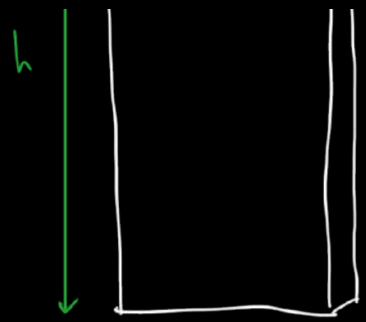
1.25" x 8.80" x 19.45" (overall)



width of wire channel: 1.25"

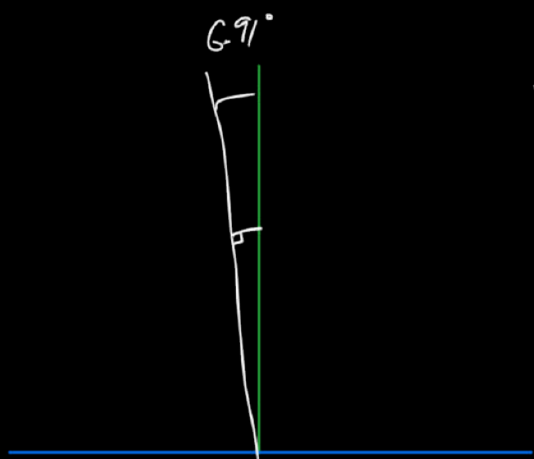
height of wire channel: 7/16" ≈ 0.44"

- horizontal layering:



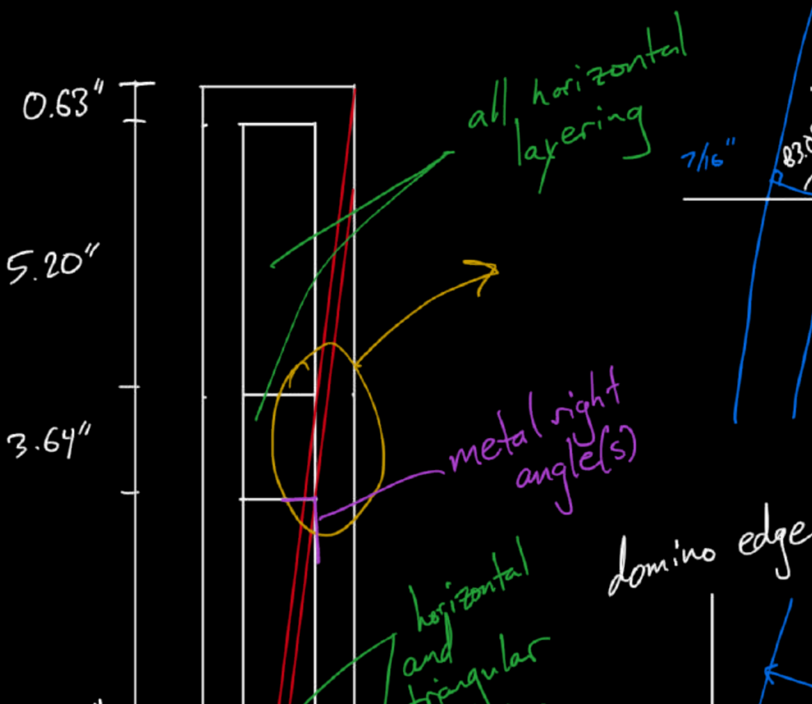
~~3 pieces, (w, h) = (2.1", 19.45")~~
~~(2 layers 1/2", 1 layer 3/16" foam board)~~

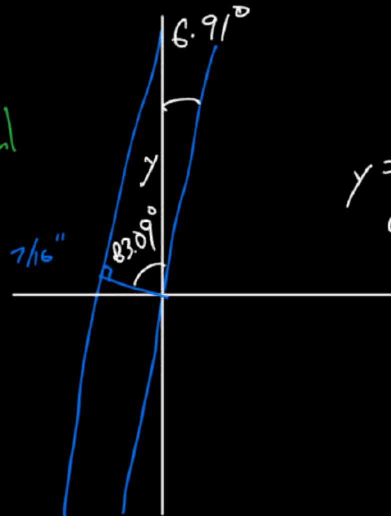
Angle between vertical and domino side is 6.91° (see other note).



$$\tan(6.91°) = \frac{0.63''}{(N \cdot 0.63'')}$$ N: # dice up side

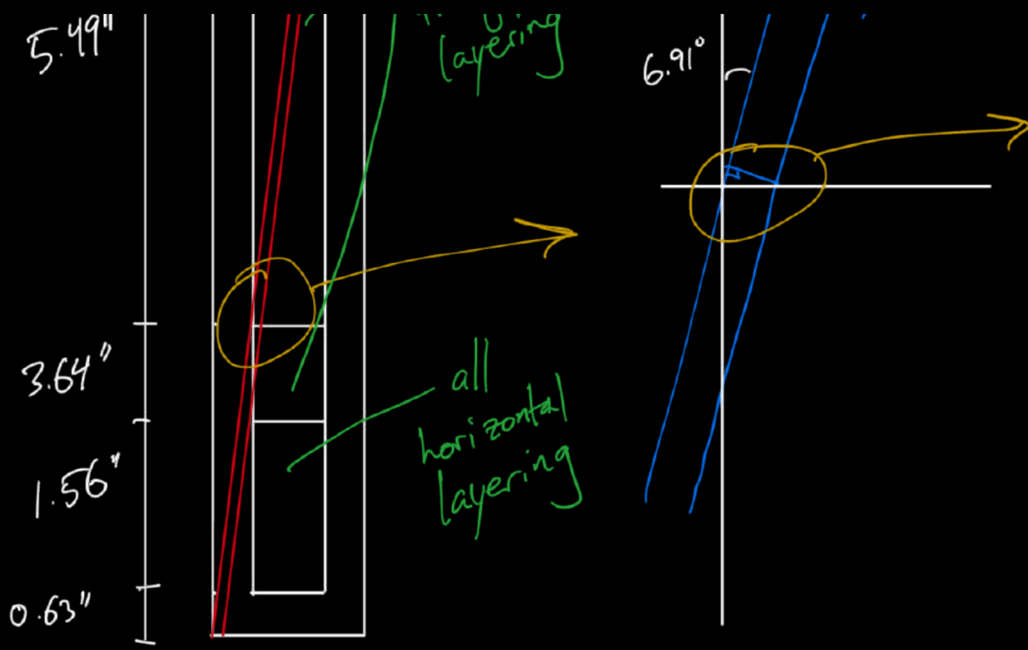
$$N = 1/\tan(6.91°) = 8.25 \text{ dice} = 5.20''$$

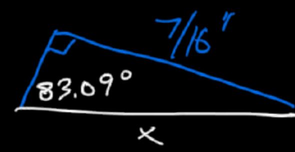




$$\cos(83.09°) = \frac{7/16''}{y}$$

$$y = \frac{7/16''}{\cos(83.09°)} = 3.64''$$

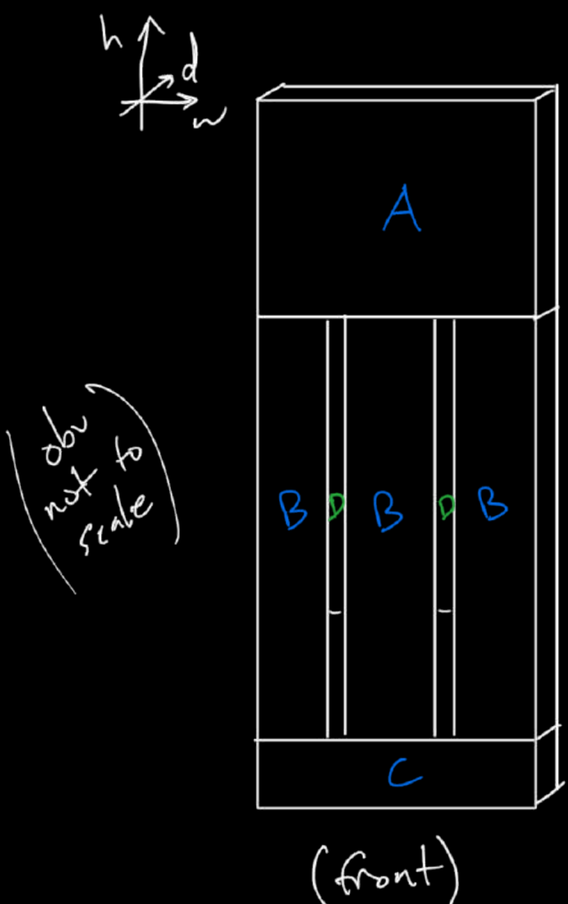




$$\sin(83.09^\circ) = \frac{7/16''}{x}$$

$$x = \frac{7/16''}{\sin(83.09^\circ)} = 0.44''$$

All of the above means:



— : horizontal layering

— : triangular layering

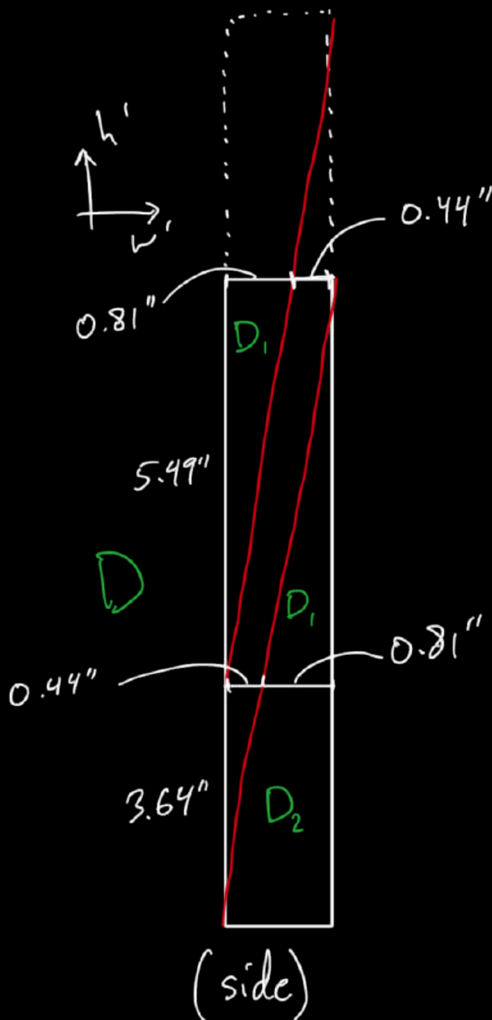
A: $(w, h) = (8.80'', 8.84'')$

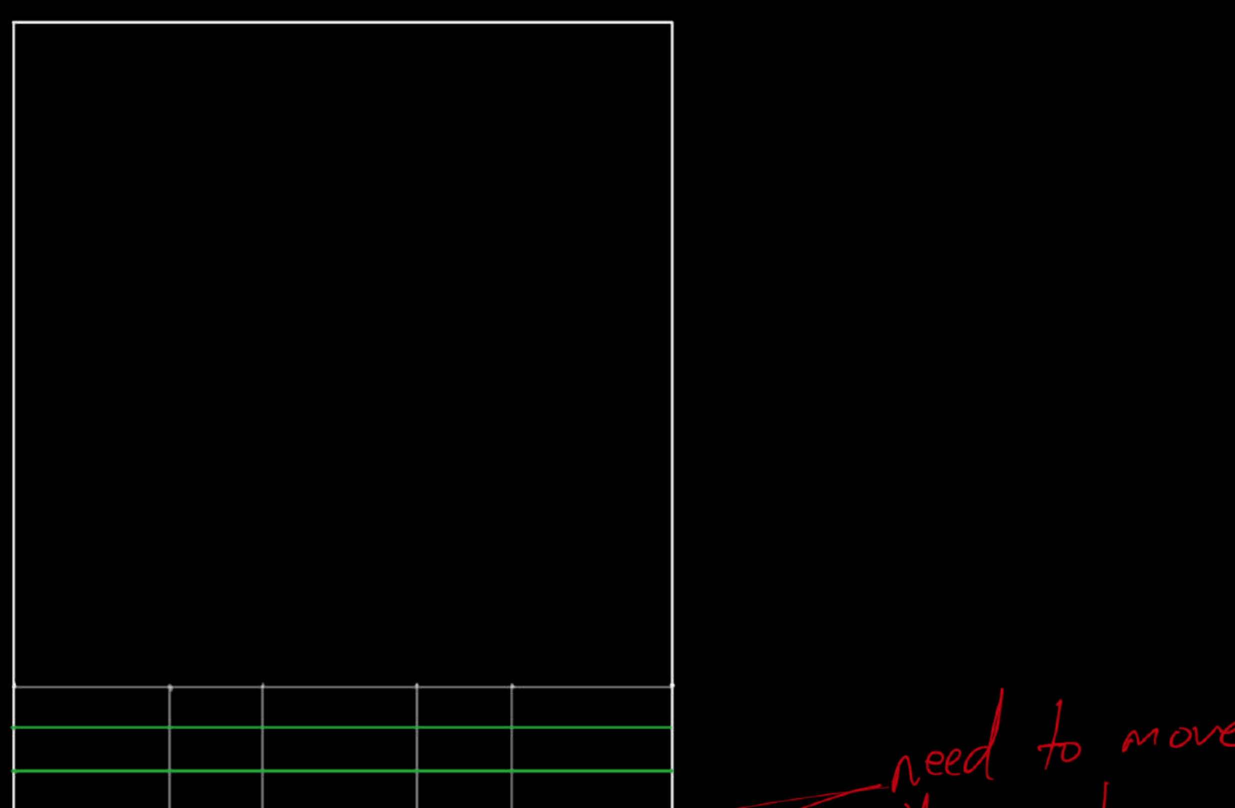
B: $(w, h) = (2.10'', 9.13'')$

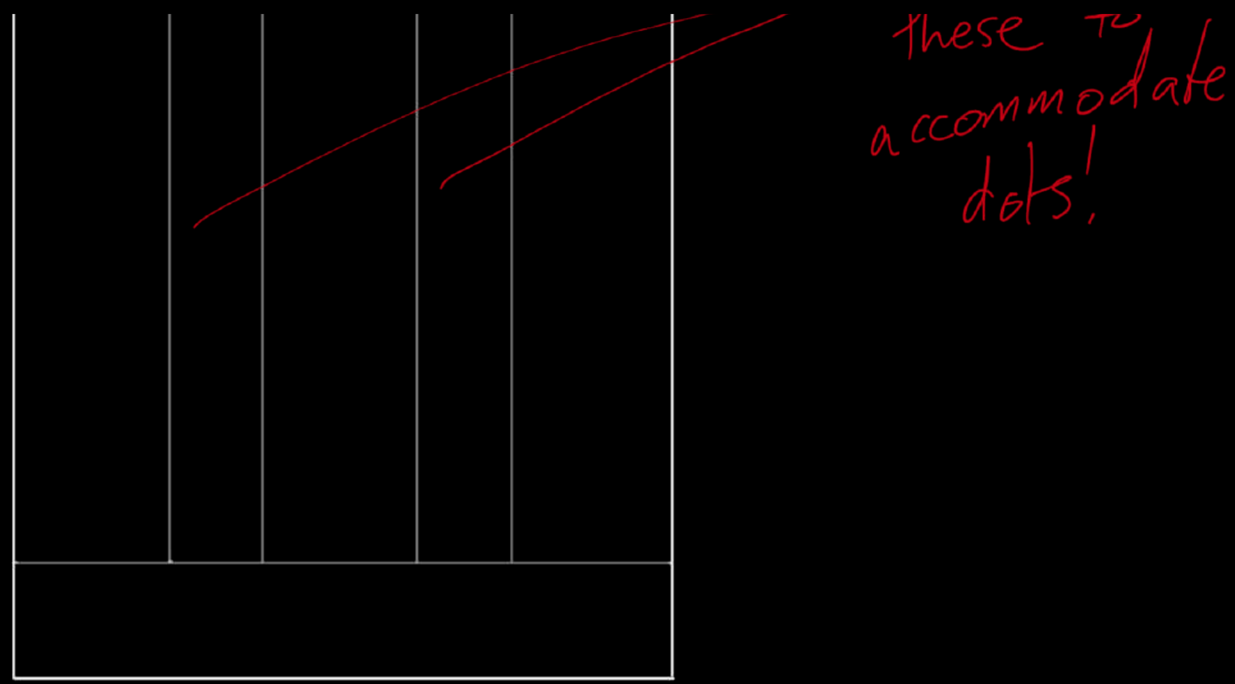
C: $(w, h) = (8.80'', 1.56'')$

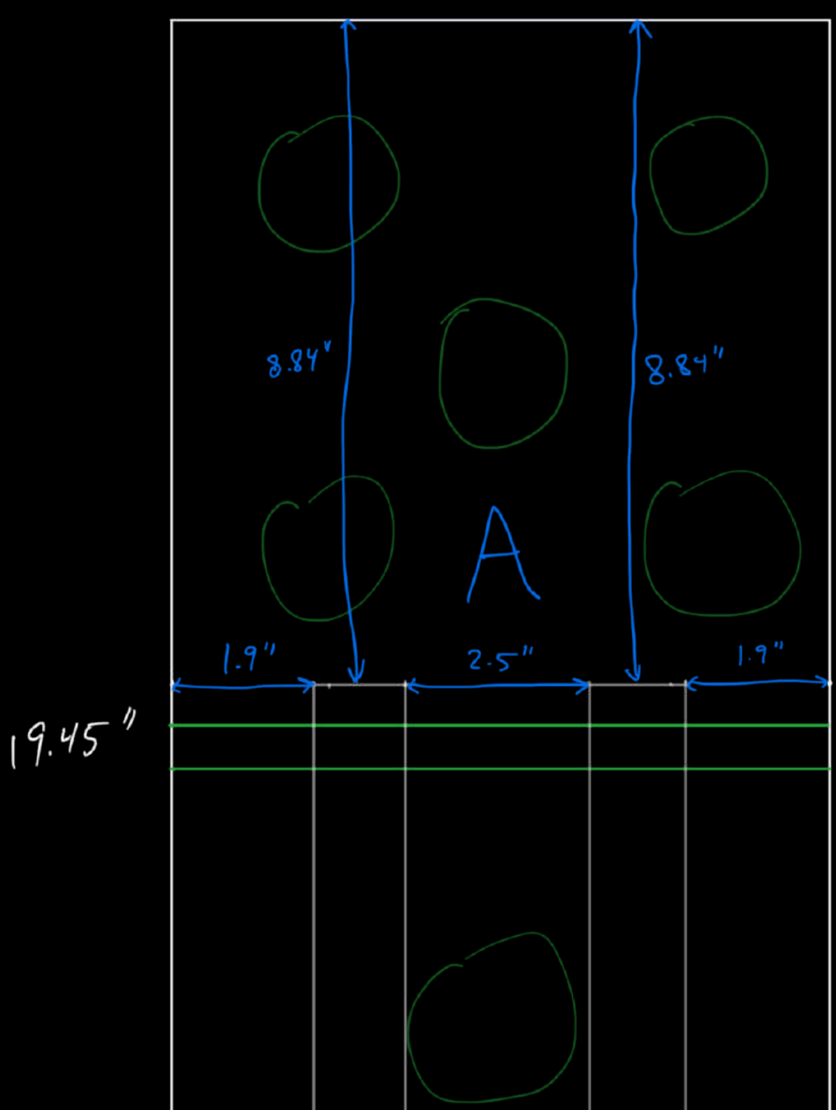
D: D_1: $(w', h') = (0.81'', 5.49'')$

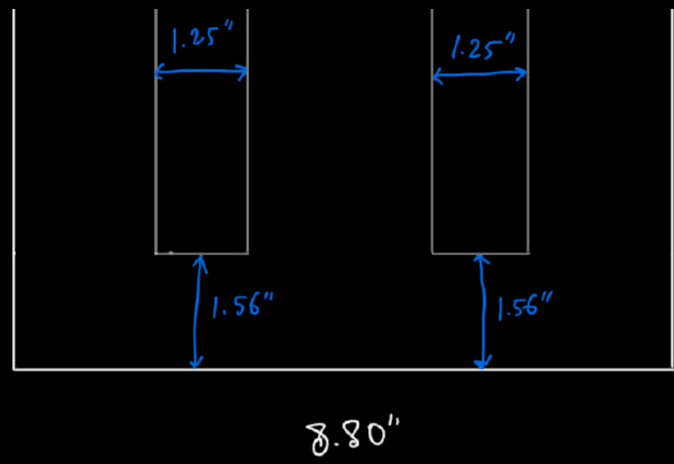
D_2: $(w', h') = (1.25'', 3.64'')$


h'
w'
0.44"
0.81"
D_1
5.49"
D
D_1
0.81"
0.44"
3.64"
D_2
(side)


need to move


these to
accommodate
dots!


8.84"
8.84"
A
1.9"
2.5"
1.9"
19.45"



Measurements on groups of actual dominoes:

- 8 end to end: 15 7/16"
- 16 side to side: 15 2/16"
- 4 end to end, 8 side to side: 15 4/16"

The Hilbert curve tiling I'm going to use has columns that vary between 2 and 6 end to end, but the bottom row has 8 end to end, so I should use that as the dimension for the square sides

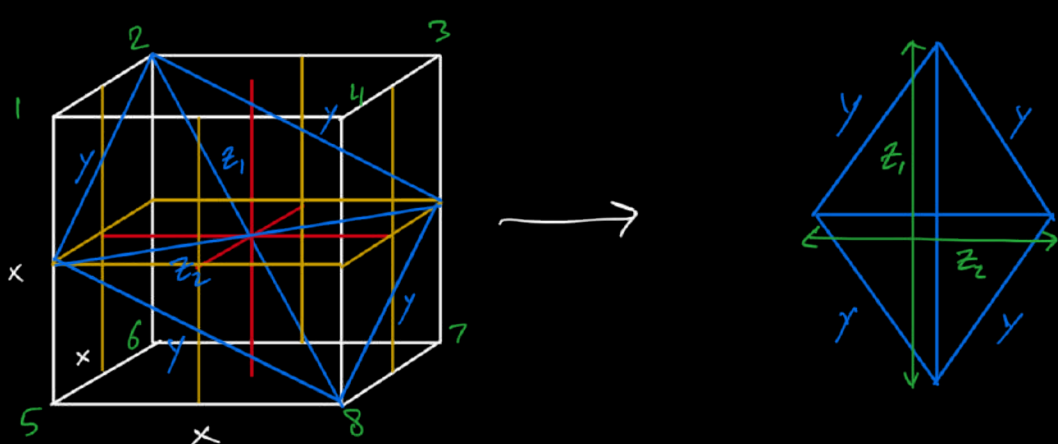
For construction of the dice of dominoes:

I'll avoid cutting 45° angles, so:

- 2 pieces $15\,7/16'' \times 15\,7/16''$ (A)
- 2 pieces $14\,7/16'' \times 15\,7/16''$ (B)
- 2 pieces $14\,7/16'' \times 14\,7/16''$ (C)

The cube is $15\,7/16'' \times 15\,7/16'' \times 15\,7/16''$.

Each side is a 16×16 grid of $0.965'' \times 0.965''$
($\approx 31/32'' \times 31/32''$)
just under this



$y^2 = x^2 + \left(\frac{1}{2}x\right)^2$ $\quad z_2^2 = x^2 + x^2$

$z_2 = \sqrt{2}\,x$

$$y = \frac{\sqrt{5}}{2}x \qquad z_1^2 = x^2 + z_2^2 \qquad z_1 = \sqrt{3}x$$

Using interior dimensions of the cube, to take foam board width into account:

$$x = 15\tfrac{7}{16}'' - 2(\tfrac{1}{2}'') = 14\tfrac{7}{16}''$$

$$y = 1.118(14\tfrac{7}{16}'') = 16.14'' \approx 16\tfrac{1}{8}''$$

$$z_2 = 1.414(14\tfrac{7}{16}'') = 20.41'' \approx 20\tfrac{13}{32}''$$

$$z_1 = 1.732(14\tfrac{7}{16}'') = 25.01''$$

OR (taking into account interior foam board too):

$$x = 15\tfrac{7}{16}'' - 2(\tfrac{1}{2}'') - 2[2(\tfrac{3}{16}'')] = 13\tfrac{11}{16}''$$

$$y = 1.118(13\tfrac{11}{16}'') = 15.30'' \approx 15\tfrac{5}{16}''$$

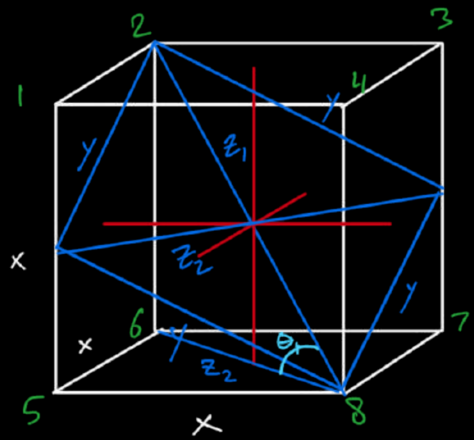
$$z_2 = 1.414(13\tfrac{11}{16}'') = 19.35'' \approx 19\tfrac{3}{8}''$$

$$z_1 = 1.732(13\tfrac{11}{16}'') = 23.71'' \approx 23\tfrac{11}{16}''$$

I wrongly assumed that a piece, like that in blue above, going from corners 2 to 8 would be perpendicular to a line going from corners 4 to 6! Oops.

It might not be an issue, but it'll make the piece not quite balanced, and it'll make ... for the metal rod

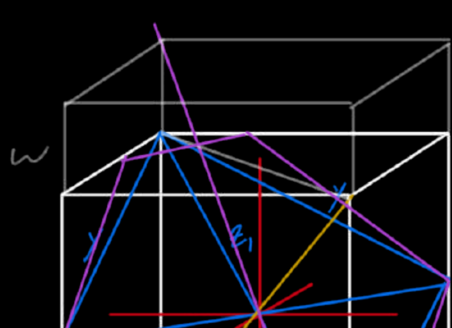
the internal plastic stopper
mount askew, also limiting the piece's stability.

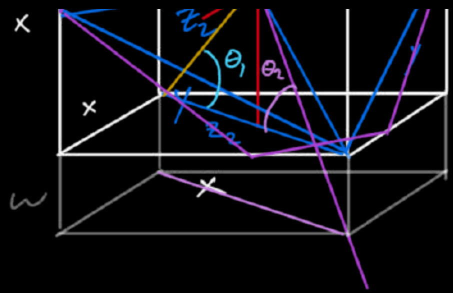


Angle of blue plane from horizontal:

$$\theta_1 = \sin^{-1}\left(x/z_1\right) = \sin^{-1}\left(x/\sqrt{3}x\right) = \sin^{-1}\left(\sqrt{3}/3\right) = 35.26°$$

That means a line perpendicular to it will be at 54.74° from horizontal. (And, equivalently, a plane would need to be at 54.74° to be ⊥ to a line at 35.26°, which is the case I'm considering.)

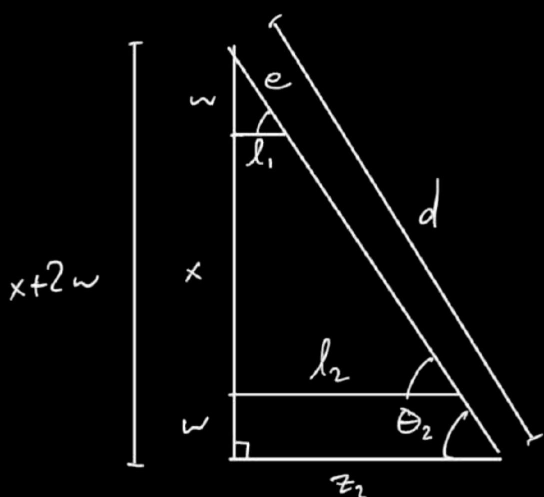




Angle of pink plane from horizontal:

$$\theta_2 = 54.74°$$

Decomposing the diagram above, you get similar triangles:



$$\cos\theta_2 = z_2/d \Rightarrow d = z_2/\cos\theta_2$$

$$= \sqrt{2}x/\cos(54.74°)$$

$$= 2.45x$$

$$(x+2w)^2 + z_2^2 = d^2 = z_2^2/\cos^2\theta_2$$

$$x^2 + 4xw + 4w^2 + z_2^2 = z_2^2/\cos^2\theta_2$$

$$x^2 + 4xw + 4w^2 + 2x^2 = 2x^2/\cos^2\theta_2$$

$$(x+2w)^2 = 2x^2\left(\frac{1}{\cos^2\theta_2} - 1\right)$$

$$x + 2w = \sqrt{2}\,x\sqrt{\frac{1}{\cos^2\theta_2} - 1}$$

$$w = \frac{1}{2}x\left(\sqrt{2\left(\frac{1}{\cos^2\theta_2} - 1\right)} - 1\right)$$

$$= \frac{1}{2}x\left(\sqrt{2(3.00 - 1)} - 1\right)$$

$$l_1 + l_2 = z_2$$

$$\frac{w}{l_1} = \tan\theta_2$$

$$l_1 = w/\tan\theta_2$$

$$= (\tfrac{1}{2}x)/\tan(54.74°)$$

$$= 0.353x$$

$$l_2 = z_2 - l_1$$
$$= \sqrt{2}x - 0.353x$$
$$= 1.06x$$

CHECK:

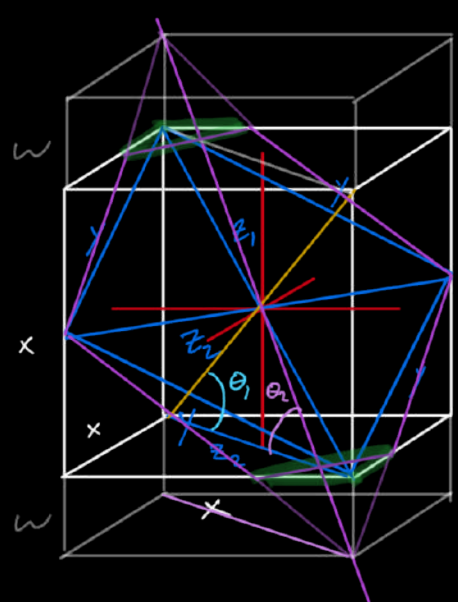
$$\frac{x+w}{l_2} = \tan\theta_2$$
$$l_2 = \frac{x+w}{\tan\theta_2}$$
$$= \frac{\frac{3}{2}x}{\tan(54.74°)}$$
$$= 1.06x \checkmark$$

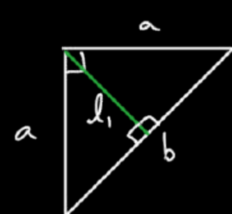
$$= \frac{1}{2}x$$

CHECK:

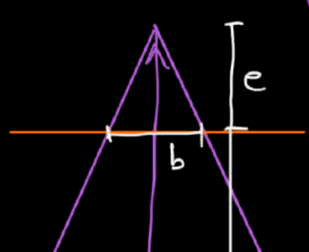
$$(x+2w)^2 + z_2^2 = d^2$$
$$(2x)^2 + (\sqrt{2}x)^2 = (2.45x)^2$$
$$6x^2 = (2.45x)^2$$
$$2.45x = 2.45x \checkmark$$

Now, working my way to these green triangles:

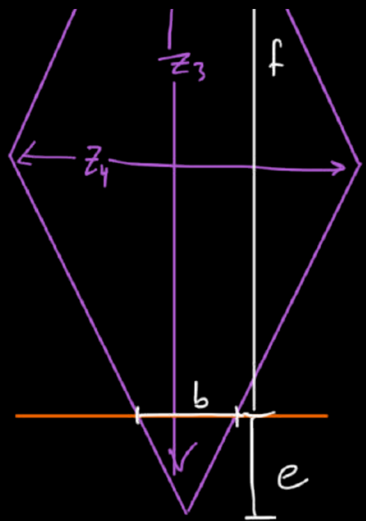




$$a = \sqrt{2l_1^2} \qquad b = 2l_1$$
$$= \sqrt{2}\,l_1$$



$$z_3 = d = \sqrt{6}x = 2.45x$$
$$z_4 = z_2 = \sqrt{2}x$$
$$e = \sqrt{l_2^2 + w^2}$$



$$= \sqrt{(0.353x)^2 + \left(\tfrac{1}{2}x\right)^2}$$

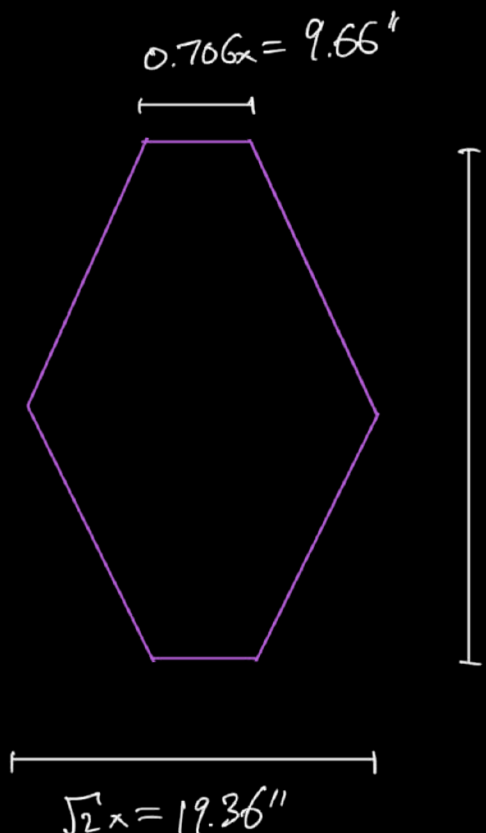
$$= 0.612x$$

$$f = z_3 - 2e = 2.45x - 2(0.612x)$$

$$= 1.23x$$

$$b = 2l_1 = 2(0.353x) = 0.706x$$

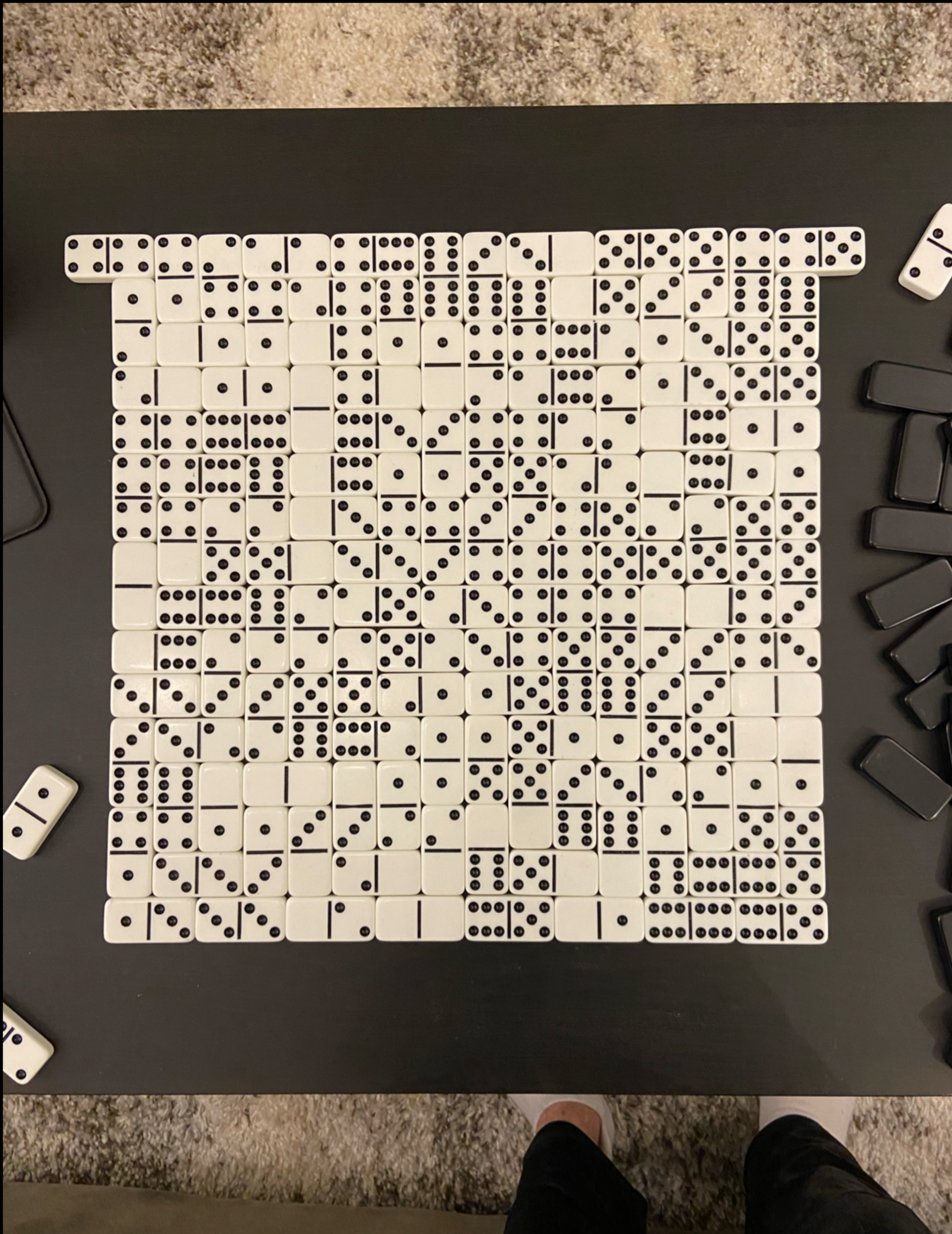
Taking (as above) $x = 13\tfrac{11}{16}''$:

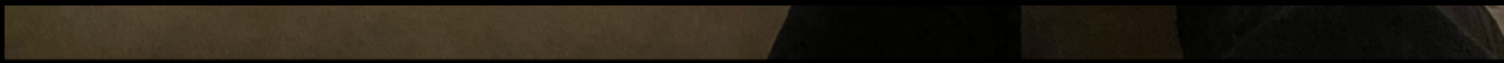


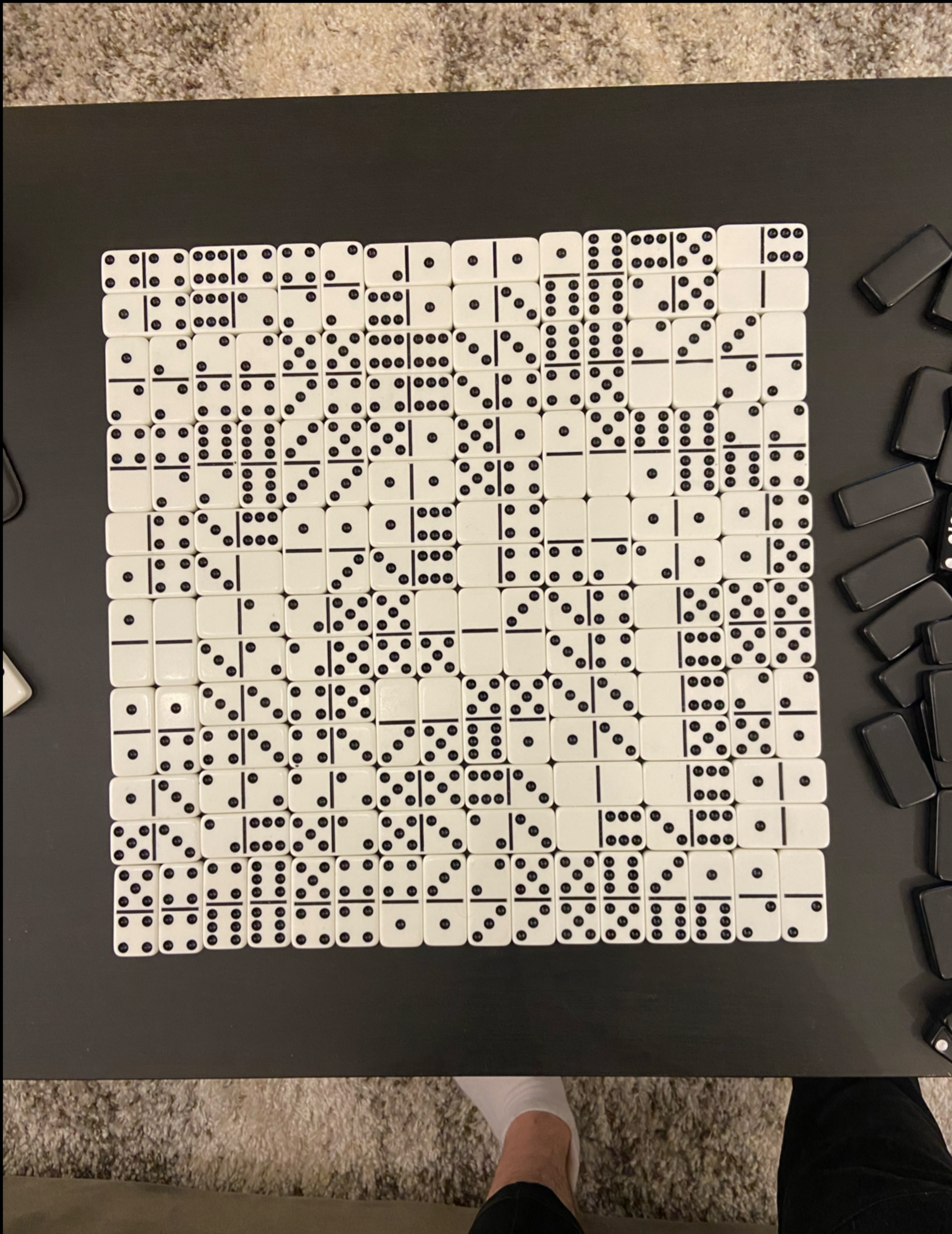
(picture obv. not to scale!!)

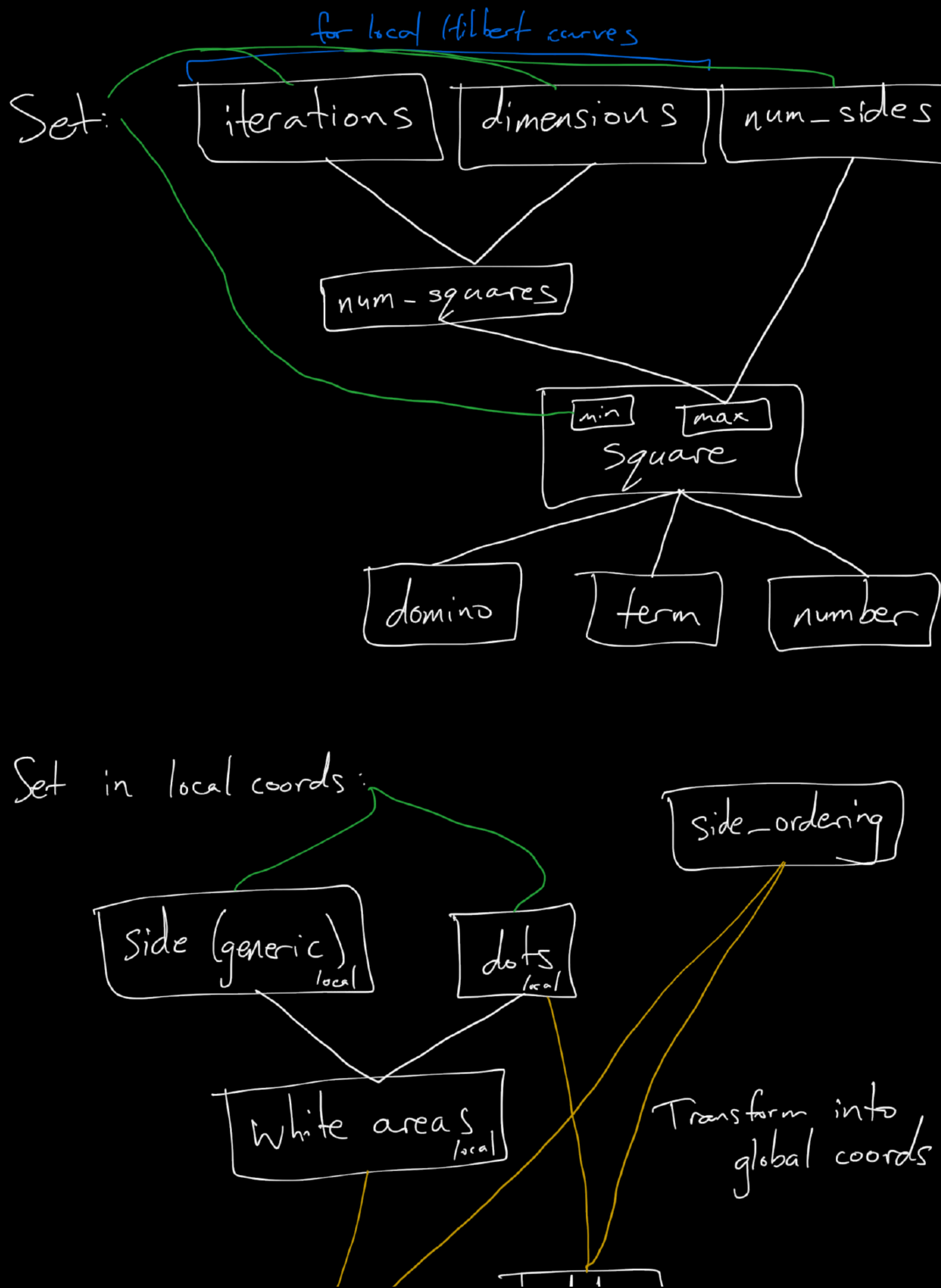
WOAH!!! This is a regular hexagon!

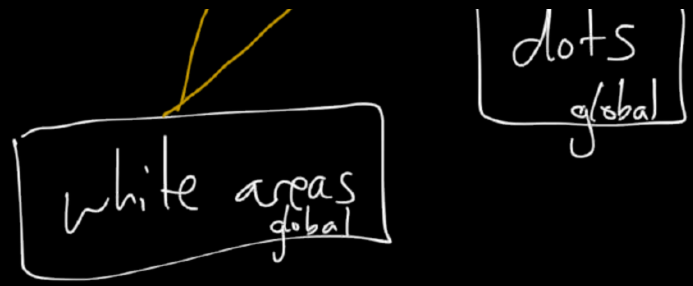
Both of the below are Hilbert curves. I like the first because it doesn't have as boring of regularities (the second's bottom row is all of the same orientation!) and because it has a more consistent mix of horizontal/vertical for each row/column so that it's closer to a perfect square.

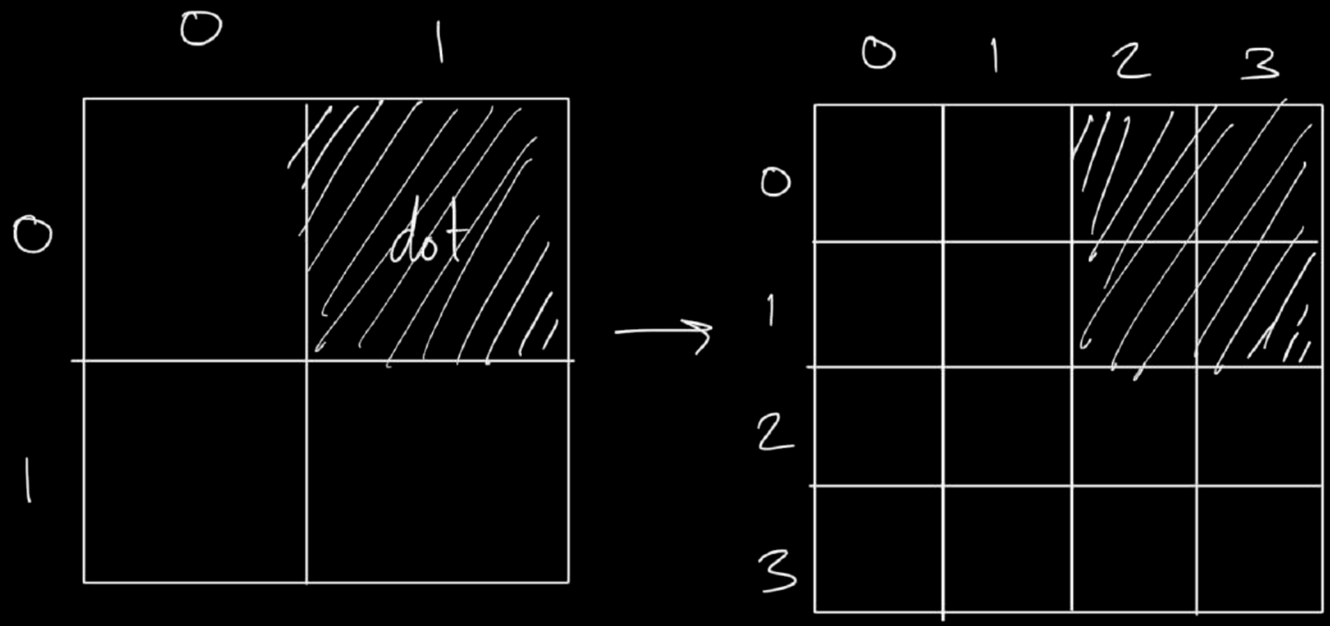




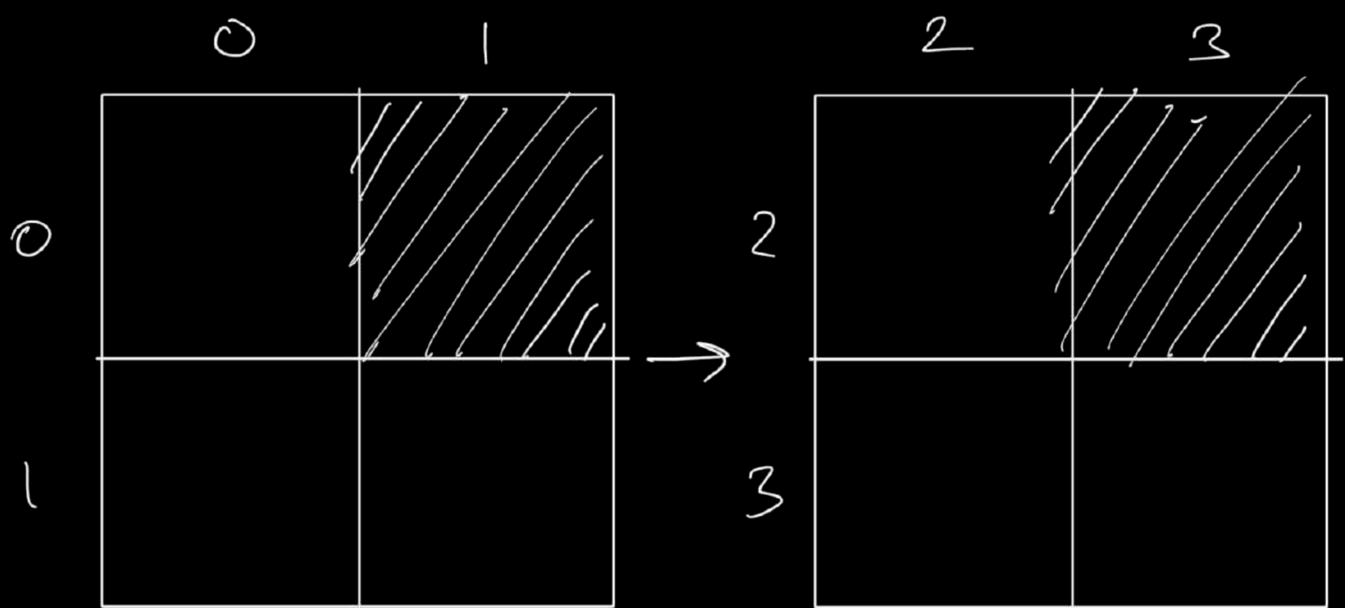



for local Hilbert curves
Set:
iterations
dimensions
num_sides
num-squares
min
max
Square
domino
term
number
Set in local coords:
side_ordering
side (generic)
local
dots
local
white areas
local
Transform into global coords


dots
global
white areas
global



dot: [[0, 1]]



dot: [[0, 1]]

dot: [[2, 3]]